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# Incorporating Multicriteria Decision Analysis Techniques in Aircraft Conceptual Design Process

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**In aerospace systems design, conflicting disciplines and technologies are always involved in the design process. There are often subjective decisions made in the conceptual design phase, and these subjective decisions have significant impacts on the performance of the final design. Multicriteria decision analysis techniques can help designers to effectively deal with such situations and make wise design decisions. The objective of this paper is to explore the feasibility and added values of applying multicriteria decision analysis techniques in aircraft design. In the first part of the paper, we establish a new optimization framework incorporating multicriteria decision analysis techniques in aircraft conceptual design process. Then, we propose an improved multicriteria decision analysis method to aggregate the multiple design criteria into one composite figure of merit for the optimization. The improved multicriteria decision analysis method is able to maintain ranking consistency for the top-ranked alternative. In the second part of the paper, we assess the subjective preferences from different designers in aircraft design process. We solve the specification problem of weighting factors by using Latin hypercube sampling (LHS) with Dirichlet distribution. Finally, we develop surrogate models for the multiple design criteria in terms of weighting factors to perform the uncertainty assessment efficiently.**

## I. Introduction

AIRCRAFT are complex systems involving multiple disciplines, among which human behavior is extremely difficult to quantify and integrate into mathematical models; more advances are needed to improve the design process for complex systems [1]. There are often subjective decisions made in the conceptual design phase, and these subjective decisions have significant impacts on the performance of the final design. The single economic criterion, such as operating cost, is not the only metric for technology evaluation as well as the figure of merit for design optimization. When using classic direct operating costs (DOC) to evaluate an aircraft, manufacturers run the risk of designing aircraft types that are not fully suited to satisfy long-term transportation needs [2,3]. In addition to the economic consideration, there are several other criteria that need to be taken into account in aircraft design and evaluation processes, such as environmental impact and level of comfort. However, it is often difficult to derive reliable transfer functions to convert these nonmonetary criteria into monetary values [4]. One solution is to apply multicriteria decision analysis (MCDA) techniques.

As an important field in operational research, MCDA is a process that allows one to make decisions in the presence of multiple, potentially conflicting criteria [5,6]. Common elements in the decision analysis process are a set of design alternatives, multiple decision criteria, and preference information representing one's attitude in favor of one criterion over another, usually in terms of weighting factors. MCDA techniques help designers to evaluate the overall performance of the design alternatives. Furthermore, MCDA techniques are helpful in the generation, analysis, and optimization of design solutions.

Although MCDA as a discipline has a relatively short history of about 40 years, several techniques have been developed to deal with different decision problems. For instance, the analytical hierarchy process (AHP) decomposes hierarchy decision problems into a series of pairwise comparisons [7]; elimination and choice translation reality (ELECTRE) methods classify candidate alternatives into nondominated alternatives and dominated alternatives [8]; preference ranking organization method for enrichment evaluations (PROMETHEE) provides a valued preference relationship among alternatives [9]; and the technique for order preference by similarity to ideal solution (TOPSIS) ranks candidate alternatives based on Euclidean distances [10].

There are typically three strategies to incorporate MCDA techniques in multicriteria decision problems: a priori approach, a posterior approach, and an interactive approach. In the a priori approach, MCDA techniques are used to aggregate the multiple design criteria into one figure of merit. Then, optimization techniques are applied to search for the most preferred design solution, with the composite figure of merit serving as a single objective function. In the a posteriori approach, optimization techniques are applied first to search for a set of nondominated solutions, usually in terms of a Pareto front. Then, MCDA techniques are used to select the most preferred design solution from the Pareto front, taking multiple evaluation criteria into consideration simultaneously. In the interactive approach, preference information is specified iteratively during the optimization process. In this paper, the a priori approach of incorporating MCDA techniques in multicriteria decision problems is further investigated, where a designer involves as a decision maker.

The a priori approach can support designers to quickly assess the compromised design alternatives and be capable of dealing with large number of objectives. One of the classical a priori approaches is the weighted-sum method, where weighting factors represent the importance of objectives and all objectives have to be normalized. The weighted-sum method has advantages in that multi-objectives are reduced to a single objective function and traditional optimization methods can be used [11]. However, the weighted-sum method suffers from the specification problem of weighting factors and the normalization method [11]. Additionally, Marler and Arora showed that the weighted-sum method provided only a linear approximation of the preferences, and the final solution may not accurately reflect one's initial preferences [12].

Furthermore, MCDA techniques suffer from a ranking inconsistency problem: the top-ranked alternative may change when an alternative is removed from or added to candidate alternatives. Belton and Gear first observed the occurrence of ranking inconsistency in AHP with the introduction of a nonoptimal alternative [13]. Keyser and Peeters discussed the behavior of ranking inconsistency in PROMETHEE, when a dominated alternative or a copy of another

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alternative was added or deleted [14]. Chen revealed that the cause of ranking inconsistency for TOPSIS lied in the determination of the two hypothetical ideal solutions [15]. Wang and Triantaphyllou investigated the ranking inconsistency in ELECTRE II and ELECTRE III, when a nonoptimal alternative is randomly replaced by a worse one [16]. Although the authors identified the occurrence of ranking inconsistency in these MCDA methods, how to mitigate or eliminate the ranking inconsistency issue was not fully addressed and solved.

In this paper, we establish a new multicriteria optimization framework incorporating MCDA techniques in aircraft conceptual design process. Then, we propose an improved MCDA method with the capability of maintaining ranking consistency. Furthermore, considering that the inherent uncertainty and subjectivity of designer's preferences have significant impact on the design solution, an uncertainty assessment that demonstrates this impact must consider different combinations of weighting factors. We solve the specification problem of weighting factors by using LHS with Dirichlet distribution. Finally, we develop surrogate models for design criteria in terms of weighting factors to perform uncertainty assessment efficiently.

The paper is organized as follows. Section II presents a literature review of applying MCDA techniques in aircraft conceptual design process. Section III proposes a new multicriteria optimization framework with an improved MCDA method. In Sec. IV, surrogate models for design criteria in terms of weighting factors are developed. Finally, conclusions are drawn in Sec. V.

## II. Literature Review

This section presents literature review of applying MCDA techniques to solve multicriteria decision problems in aircraft conceptual design process.

Bandte developed a probabilistic MCDA method for multicriteria optimization and product selection [17]. With the combination of multicriteria and probabilistic design, this method estimates the probability of satisfying the criteria simultaneously. However, this method does not consider the absolute location of joint probability distribution [18]. Kirby used TOPSIS for the selection of technology alternatives in aircraft conceptual and preliminary design [19]. However, the top-ranked alternative probably becomes inconsistent when candidate alternatives are changed. Li developed a multicriteria interactive decision-making advisor for the selection of the most appropriate MCDA method [20]. However, only a few methods were implemented, and uncertainty propagation was not addressed explicitly. More recently, Lan et al. proposed a web-based computer-aided system for airframe material selection in the aircraft design process [21]. Messai et al. presented a decision support approach based on lattice structures of the data in complex system design process, with the application to the ventilation system in aircraft cabin design process [22]. Sullivan et al. investigated multicriteria optimization using discrete and continuous design decision support visualization schemes [23].

The modeling and incorporation of human subjectivity in conceptual design process can help to obtain preferred designs. Cvetkovic discussed decision support methods in engineering conceptual design and developed a preference method, which translates vague qualitative information into quantitative values based on fuzzy preferences and graph theory [11]. Tappeta et al. developed an interactive physical programming framework that takes into account designer's preferences during the optimization process [24]. Gurnani and Lewis proposed to use bounded rationality to improve solutions for convergent decentralized design problems [25]. Hunt et al. used matrices to model designer's preferences in bicriteria design problems [26]. Huang and Bloebaum considered designer's preferences by incorporating specific objective ranges and targets during the optimization process [27]. Barnum and Mattsony presented a computationally assisted methodology for incorporating designer's preferences into successive iterations of design concepts [28].

From these applications of MCDA techniques in multicriteria decision problems, two observations can be formulated. First, direct application of existing MCDA techniques in the design process may suffer from a ranking inconsistency problem when candidate alternatives are changed. Second, it is crucial to model and incorporate designer's preferences in the design process. Therefore, this research investigates how to improve existing MCDA techniques to maintain ranking consistency and how to model designer's preferences in aircraft design process.

## III. New Multicriteria Optimization Framework

In this research, we establish a new multicriteria optimization framework incorporating MCDA techniques in aircraft conceptual design process, as illustrated in Fig. 1. We propose an improved MCDA method to aggregate multiple design criteria into one composite figure of merit for the optimization. The improved MCDA method is able to maintain ranking consistency when candidate alternatives are changed. The proposed optimization framework can support designers to quickly assess the compromised design alternatives, which is especially valuable in aircraft conceptual design stage.

We apply the proposed optimization framework to the design of a conventional 150 passenger, twin-engine airliner with a design range of 3200 km, using the conceptual aircraft design tool VAMPzero (from "virtual aircraft multidisciplinary analysis and design processes"). VAMPzero is developed at DLR, German Aerospace Center and is licensed under the Apache 2.0 license [29].

In this section, we first discuss the identification of multiple design criteria. Then, we propose an improved MCDA method for the established multicriteria optimization framework. Last, we present the optimization results for typical weighting scenarios.

### A. Identification of Design Criteria

Selection of appropriate design criteria is critical to the determination of an optimal design. Raymer provided some recommendations [30]: the design criterion should represent a nontrivial and calculable indication of the worth of the concept; it should be significantly affected by the design variables and constraints; it should have clear meaning to designers and customers; and it needs clear rationale for methods and factors used for blending if it is blended.

In this study, to explore the interrelationships among the interest of manufacturers, the concern of fuel-based emissions, the concerns of airliners, and the consideration of passenger comfort explicitly, four design criteria are selected to feed into the MCDA method: operating empty mass (OEM), fuel mass, utilization/block time, and passenger density.

Annual utilization is the number of flight hours actually flown annually by an aircraft or by a fleet of aircraft, while annual utilization rate is a percentage of the flight hours actually flown relative to the maximum theoretical available hours. In this paper, annual utilization for an aircraft is further investigated. Table 1 explains how we estimate the maximum operation hours for an aircraft [31]. Equation (1) shows annual utilization for an aircraft when we take

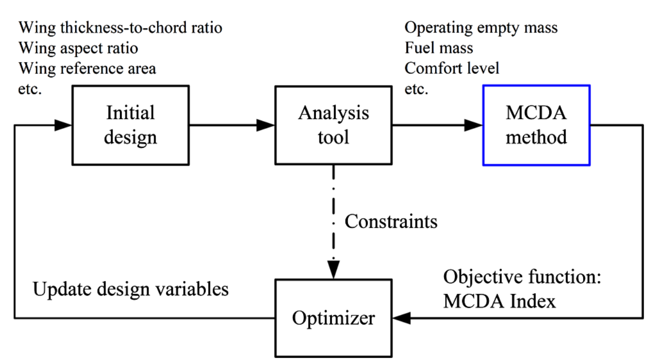


**Fig. 1 Framework of incorporating MCDA techniques in aircraft design process.**

Table 1 Estimation of the maximum operating hours [31]

| Maximum operating hours per year | 365 days * 24 h | 8760 h/year |
|---|---|---|
| Six days operation in a week | −52 days * 24 h | −1248 h/year |
| Six weeks failure due to maintenance | −36 days * 24 h | −864 h/year |
| Compliance with airport night curfews (23:00–6:00) | −277 days * 7 h | −1939 h/year |
| Compliance with route plan | | −511 h/year |
| Available operating hours | | 4198 h/year |

into account its turnaround time [31]. Here, we assume that the aircraft is grounded for a quarter of an hour to an hour (15 min taxi time plus 30 min turnaround time, thus, in total, 45 min, or 0.75 h).

Block time calculates the time from engines on to engines off for the design mission [32]. When dividing both sides of Eq. (1) with block time, we can get the mathematical expression of annual utilization/block time, as shown in Eq. (2). The ratio of utilization/block time provides a rough estimation about the number of flight per year for an aircraft. Passenger density is defined as the number of passenger seats divided by cabin base area, where cabin base area is calculated as the product of fuselage diameter and cabin length. Its mathematical formula is given in Eq. (3).

Cabin length in Eq. (3) is an output parameter and is calculated by Eq. (4) [33], where class layout factor is 1 for a single class layout, and range type factor is 0.9 for a short-range aircraft type. The number of passenger abreast in one row is calculated from the total number of passengers in the aircraft. In the case of 150 passengers, it is equal to 6. The aircraft in the example analysis is a single aisle with six abreast configuration:

$$\text{Annual Utilization} = \frac{4198}{1 + \frac{0.75}{\text{Block time}}} \tag{1}$$

$$\text{Utilization}/(\text{Block time}) = \frac{4198}{0.75 + \text{Block time}} \tag{2}$$

$$\text{Passenger Density} = \frac{\text{Number of passenger seats}}{\text{Fuselage diameter} * \text{Cabin length}} \tag{3}$$

$$\text{Cabin length} = \frac{\text{Number of passenger seats} * \text{Class layout factor}}{\text{Range type factor} * \text{Number of passenger abreast}} \tag{4}$$

Attention should be paid that Eq. (1) for annual utilization does not consider that the aircraft fly complete missions per day. From Table 1, we can observe that an aircraft has 277 days for operation, considering that the aircraft is operated six days per week and has six weeks downtime for maintenance ($365 - 52 - 36 = 277$). Within these 277 days, the aircraft is allowed to be operated a maximum of 17 h due to 7 h of airport night curfews. The number of complete missions that can be flown per day is approximated by 17/(block time+ turnaround time). In this conceptual aircraft example, the design range is 3200 km, and the cruise Mach number is between 0.70 and 0.84. The complete missions for an aircraft per day with lower speed can be estimated by $3200/(0.7 * 340 * 3.6) \approx 3$, while with higher speed can be estimated by $3200/(0.84 * 340 * 3.6) \approx 4$. The average data approach using annual utilization in the current research might not reflect if there would be an abrupt change point with speed. Although this average data approach has less influence on the performance evaluation for long range aircraft, it does have more influence for short- and medium-range aircraft. This issue could be addressed in future research.

The focus of the proposed optimization framework is to assess the added values of incorporating MCDA techniques in aircraft conceptual design process. Thus, to keep the design process transparent, the complexity of the design problem is limited. We consider five design variables for a conceptual aircraft design model: wing thickness-to-chord ratio, wing aspect ratio, wing reference area, cruise Mach number, and fuselage diameter. The constraints imposed in the aircraft design process are wing span, fuel tank volume, takeoff field length, landing field length, takeoff wing loading, and cruise thrust. The design variables, constraints, and design criteria for this simplistic aircraft design model are summarized in Table 2.

Table 2 Summary of design variables, constraints, and design criteria in aircraft optimization process

| Parameter | Value |
|---|---|
| *Design variables* | |
| Wing thickness-to-chord ratio | [0.1, 0.2] |
| Wing aspect ratio | [8, 12] |
| Wing reference area, $m^2$ | [80, 140] |
| Cruise Mach number | [0.70, 0.84] |
| Fuselage diameter, m | [3.8, 4.2] |
| *Constraints* | |
| Wing span, m | ≤36 |
| Fuel mass, kg | ≤Fuel density * fuel tank volume |
| Takeoff field length, m | ≤3000 |
| Landing field length, m | ≤2000 |
| Takeoff wing loading, $kg/m^2$ | ≤600 |
| Cruise thrust, N | ≤0.9 Takeoff thrust |
| *Design criteria* | |
| OEM, kg | — — |
| Fuel mass, kg | — — |
| Utilization/block time | — — |
| Passenger density, passenger/$m^2$ | — — |

This problem formulation represents a simplistic aircraft design model; an extended version that includes more design variables and constraints could be analyzed in future research. The two nonmonetary criteria of fuel mass and passenger density demonstrate how to incorporate intangible design criteria in a multicriteria optimization framework. Other representations of intangible design criteria are of future interest.

### B. Improved Technique for Order Preference by Similarity to Ideal Solution Method

After the identification of the multiple design criteria, we propose an improved MCDA method to aggregate the multiple design criteria into one composite figure of merit for the optimization. TOPSIS is recommended by a multicriteria decision support system as the most appropriate one to solve this aircraft design decision problem [34]. In the TOPSIS method [5], two ideal solutions are hypothesized: a positive ideal solution, which has all of the best criteria values, and a negative ideal solution, which has all of the worst criteria values. TOPSIS selects the alternative that is closest to the positive ideal solution and farthest from the negative ideal solution.

For the purpose of illustration, we can imagine that TOPSIS puts the alternatives into a coordinate system. For example, if there are three criteria, it is a three-dimensional coordinate system, as shown in Fig. 2, where the dot $A+$ represents the positive ideal solution, and the dot $A-$ represents the negative ideal solution. TOPSIS ranks the alternatives based on the Euclidean distance to these two ideal solutions.

However, as discussed in Sec. I, the TOPSIS method suffers from a ranking inconsistency problem. When an alternative is removed from or added to the candidate alternatives, the ideal solutions will probably change, and the Euclidean distances to the ideal solutions will also change. Thus, the top-ranked alternative may become inconsistent when the candidate alternatives are changed.

In this study, an improved TOPSIS (ITOPSIS) is proposed to aggregate the four design criteria into one figure of merit for the optimization. The positive ideal solution and negative ideal solution are calculated and set beforehand to maintain ranking consistency. In this example, the calculation of these ideal solutions are obtained through single objective optimizations for each of the four design criteria, as shown in Fig. 3.

For instance, to find the ideal solutions for fuel mass, we conduct minimization and maximization for fuel mass, respectively. The minimum value of fuel mass serves as the positive ideal solution, while the maximum value of fuel mass serves as the negative ideal solution. The ideal solutions for the other three criteria are specified

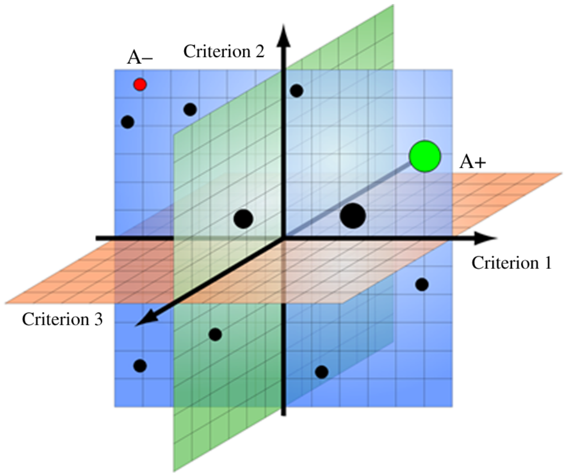


**Fig. 2 TOPSIS in three-dimensional coordinate system.**

in a similar way. These ideal solutions for the four design criteria are summarized in Table 3. It should be noted that higher value of utilization/(block time) ratio is preferred, while lower values of the other three design criteria are preferred.

### C. Optimization Results with Typical Weighting Scenarios

In previous sections, we identified four design criteria and proposed an improved MCDA method (ITOPSIS) to aggregate the four design criteria into one composite figure of merit for the optimization. In this section, we investigate several typical weighting scenarios in the optimization process, ranging from one single criterion to equally preferred four design criteria. This is one approach to simulate designers' preferences. In this example, based on parametric studies, all design variables under investigation are continuous, and objective functions with respect to design variables in the conceptual aircraft design tool (VAMPzero) are rather smooth. Therefore, gradient-based methods are used in the optimization framework. Optimization results for a single criterion are summarized in Table 4, and optimization results with equal weighting factors among the four design criteria are summarized in Table 5, respectively.

It can be seen from Table 4 that, when optimizing OEM, the fuselage diameter is reduced to the lower boundary, the aspect ratio is reduced by 14%, the reference area is decreased by 5%, and the thickness-to-chord ratio is increased by 21%. The decrease of the aspect ratio and reference area leads to a reduction in wing weight, which contributes to a reduction in OEM. However, the decrease of the aspect ratio and reference area and the increase of the thickness-to-chord ratio result in an increment of the overall drag of the aircraft and 9% reduction in the cruise Mach number. The reduction in the cruise Mach number leads to a 5% decrease in utilization/block time.

**Table 3 Positive ideal solution and negative ideal solution in ITOPSIS**

| Ideal solutions | OEM | Fuel mass | Utilization/block time | Passenger density |
|---|---|---|---|---|
| Positive | 36,943 | 11,767 | 796.86 | 1.2875 |
| Negative | 50,521 | 20,864 | 715.08 | 1.4063 |

Besides, the decrease of the fuselage diameter leads to a 5% increase in the passenger density.

When optimizing the aircraft for fuel mass, the aspect ratio is increased by 24%, the reference area is increased by 8%, and the thickness-to-chord ratio is decreased by 6%. The increase of the aspect ratio and reference area leads to a larger span and an increase in wing weight, which further leads to the increase of OEM. Flying slower (low cruise Mach number) can also reduce fuel consumption for certain mission ranges. However, lower cruise Mach number prolongs block time, and the utilization/block time ratio is decreased. Because the overall drag of the aircraft can be reduced when the wetted area of fuselage is reduced, fuselage diameter is decreased to the lower boundary in the optimization process of fuel mass.

When optimizing the aircraft for utilization/block time, the cruise Mach number is increased to the upper boundary, the fuselage diameter is reduced so that the wet area of fuselage is reduced, and the reference area is increased by 5%. The decrease of the fuselage diameter and the increase of the reference area lead to the reduction of the overall drag of the aircraft. However, the increase of the cruise Mach number will burn more fuel for a specific mission range; thus, fuel mass is increased by 19%. The increase of the reference area leads to the increase of OEM. Besides, the decrease of the fuselage diameter results in a 4% increase in the passenger density.

When optimizing the aircraft for passenger density, the fuselage diameter is increased to its upper limit. The reference area is increased slightly by 3%; the thickness-to-chord ratio, aspect ratio, and cruise Mach number almost do not change. Although the utilization/block time ratio has decreased slightly, all other criteria have been increased by around 2.5%.

The conflicting design criteria are further explored when weighting factors are evenly distributed, as summarized in Table 5. The thickness-to-chord ratio is increased by 4%, the aspect ratio almost does not change, the reference area is decreased by 4%, the cruise Mach number is decreased by 2.5%, and the fuselage diameter is decreased to its lower boundary. The reduction of OEM and fuel mass is compromised by the decrease of utilization/block time ratio and the increase of passenger density.

Considering the crucial impact of designer's preferences on the optimized design, the roles of designer's preferences in the multicriteria optimization framework will be further investigated in the following section.

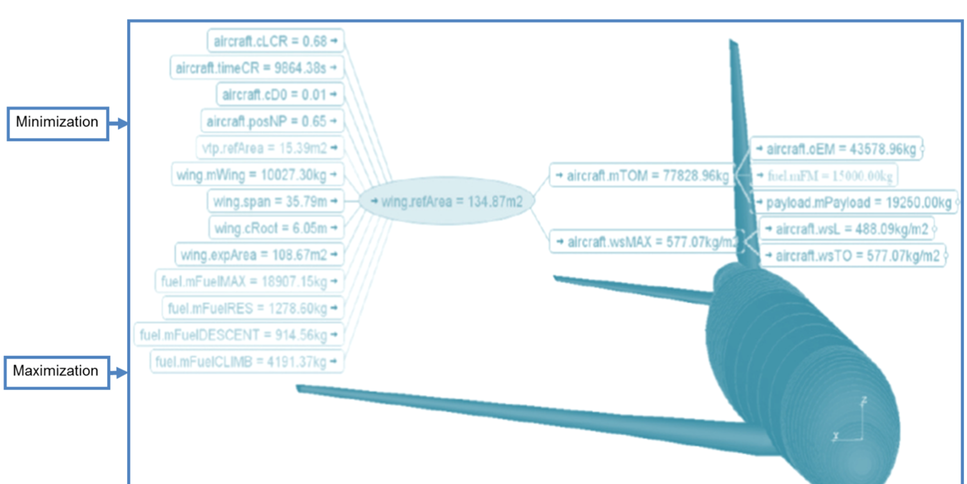


**Fig. 3 ITOPSIS in aircraft conceptual design process.**

**Table 4 Optimization results for single criterion**

| | Baseline design | Minimum OEM | Minimum fuel mass | Maximum utilization/block time | Minimum passenger density |
|---|---|---|---|---|---|
| | | | *Design variables* | | |
| Wing thickness-to-chord ratio | 0.13 | 0.1585 | 0.1220 | 0.1286 | 0.1301 |
| Wing aspect ratio | 9.4 | 8.0347 | 11.6740 | 9.3237 | 9.3608 |
| Wing reference area, $m^2$ | 122.40 | 116.18 | 132.05 | 128.53 | 125.77 |
| Cruise Mach number | 0.78 | 0.71 | 0.73 | 0.84 | 0.77 |
| Fuselage diameter, m | 4 | 3.8 | 3.8 | 3.9 | 4.2 |
| | | | *Design criteria* | | |
| OEM, kg | 40,980 | 36,949 | 43,725 | 42,974 | 42,426 |
| Fuel mass, kg | 12,903 | 13,280 | 11,771 | 15,319 | 13,312 |
| Utilization/block time | 763 | 722 | 734 | 797 | 759 |
| Passenger density, passenger$/m^2$ | 1.35 | 1.4211 | 1.4211 | 1.3863 | 1.2981 |

**Table 5 Optimization results with equal weighting factors**

| | Baseline design | Optimized design | Relative change |
|---|---|---|---|
| | *Design variables* | | |
| Wing thickness-to-chord ratio | 0.13 | 0.135 | 3.84% |
| Wing aspect ratio | 9.396 | 9.414 | 0.19% |
| Wing reference area, $m^2$ | 122.4 | 117.01 | −4.40% |
| Cruise Mach number | 0.78 | 0.76 | −2.55% |
| Fuselage diameter, m | 4 | 3.8 | −5% |
| | *Design criteria* | | |
| OEM, kg | 40,980 | 38,705 | −5.55% |
| Fuel mass, kg | 12,903 | 12,242 | −5.12% |
| Utilization/block time | 763 | 752 | −1.53% |
| Passenger density, passenger$/m^2$ | 1.35 | 1.4211 | 5.26% |



## IV. Surrogate Model Development for Design Criteria

Having established the multicriteria optimization framework shown in Fig. 1 in Sec. III, we now develop surrogate models for design criteria in terms of weighting factors. As one way to represent designer's preference information, weighting factors create a compound figure of merit for the optimization. Different weighting schemes result in different compound figures of merit. The selection of figure of merit is critical to the determination of an optimal design because, if a design is optimized to the wrong figure of merit, it will not be the best design in terms of the real important measure. An uncertainty assessment that demonstrates this crucial impact on the design solution considers different combinations of the weighting factors.

In this research, Monte Carlo simulation is used to imitate the designer's preference information among the design criteria. However, the computation time for one set of weighting factors is at least 5 min. A Monte Carlo-based uncertainty analysis with 1000 samples takes at least 84 h. The long computation time makes the uncertainty assessment an expensive computational task.

In this study, to perform the uncertainty assessment efficiently, surrogate models for the four design criteria in terms of weighting factors are developed. Each point of this surrogate model represents an optimized aircraft design for a given set of weighting factors. The whole multicriteria optimization framework is treated as a black box. An overview of surrogate modeling process for design criteria in terms of weighting factors is shown in Fig. 4.

There are typically four steps in surrogate model building process: sample the design space using experimental design, choose a model to represent the input and output data, select a method to fit the model, and validate the constructed model [35]. The surrogate model development for design criteria in terms of weighting factors will follow this process.

### A. Experimental Design

To explore the design space thoroughly, an experimental design with spatially uniform distribution is one effective approach. There are several space-filling strategies [36], among which LHS is one reliable method to generate random candidate samples, with the guarantee that these samples are relatively uniformly distributed in the design space [37].

In this study, the weighting factors $\{w_1, w_2, \ldots, w_n\}$ generated by the experimental design have to satisfy two conditions: 1) $0 \le w_i \le 1$, and 2) $\sum_{i=1}^{n} w_i = 1$. Standard LHS meets condition 1, which states that all of the factor settings range from 0 to 1. However, for each experimental run, the sum of the factor settings do not equal 1. In this research, to generate experimental designs fulfilling conditions 1 and 2, the standard LHS is conducted first, and then the samples generated by LHS are rectified by Dirichlet distribution.

In the following sections, we first introduce the modified LHS with Dirichlet distribution, and then we present one example to compare standard LHS, normalized LHS, and the modified LHS with Dirichlet distribution.

#### *1. Modified Latin Hypercube Sampling with Dirichlet Distribution*

Dirichlet distribution is a family of continuous multivariate probability distributions parameterized by a vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_k,)$ of positive reals. Dirichlet distribution is one multivariate generalization of the beta distribution and is defined as Eq. (5):

$$\mathrm{Dir}(X, \alpha) = \frac{\Gamma(\alpha_1 + \alpha_2 + \ldots + \alpha_k)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\ldots\Gamma(\alpha_k)} \prod (x_1^{\alpha_1 - 1} x_2^{\alpha_2 - 1} \ldots x_k^{\alpha_k - 1}) \quad (5)$$

where $X = (x_1, x_2, \ldots, x_{k-1})$, satisfying $x_i > 0$ and $\sum_{i=1}^{k-1} x_i < 1$. Besides, $x_k = 1 - x_1 - x_2 - \ldots - x_{k-1}$. In a symmetric Dirichlet

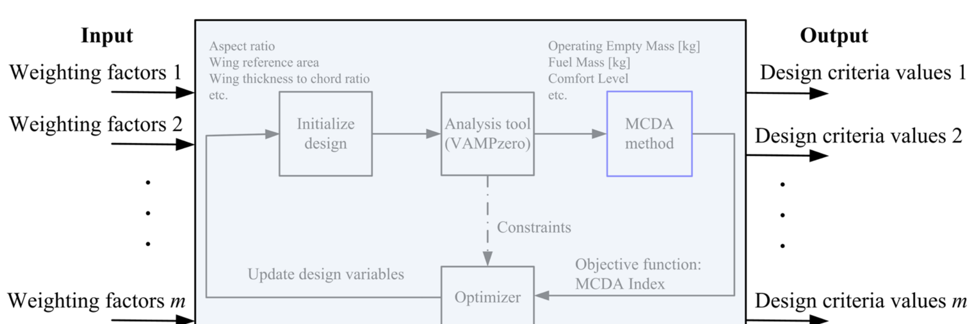


**Fig. 4 Overview of surrogate modeling process for design criteria.**

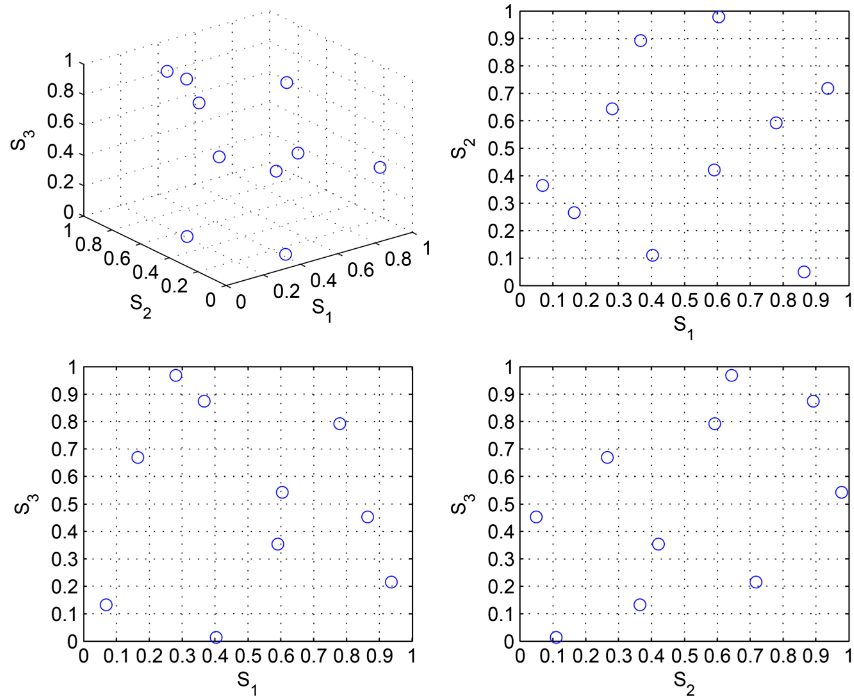


**Fig. 5 Standard LHS in three dimensions and with two-dimensional projections.**

distribution, the components of vector $\alpha$ are equal. If each component of $\alpha$ is 1, then the symmetric Dirichlet distribution is equivalent to a uniform distribution; if each component of $\alpha$ is bigger than 1, then it prefers dense, evenly distributed distribution; and if each component of $\alpha$ is smaller than 1, then it prefers sparse distribution.

### 2. *Example of Standard, Normalized, and Modified Latin Hypercube Sampling with Dirichlet Distribution*

We present one example of standard LHS, normalized LHS, and the modified LHS with Dirichlet distribution. To generate 10 sets of weighting factors for three criteria, standard LHS is conducted first, as shown in Fig. 5, where $S_1$, $S_2$, and $S_3$ represent the sample values for the three criteria. It is noted that there is exactly one point in each row and each column in the two-dimensional projections, and the sample values range from 0 to 1 (which meets condition 1); however, the sum of one set of the sample values is not equal to 1 (which does not meet the condition 2).

Thus, to fulfill condition 2, standard LHS can be normalized by its row sum, as shown in Fig. 6, where $Lw_1$, $Lw_2$, and $Lw_3$ represent the normalized sample values for the three criteria. We can observe that the range of the normalized sample values shrinks into 0 to 0.8. Moreover, there is no point in the bins bigger than 0.8; thus, the hypercube is deformed, and the Latin properties are not maintained.

The modified LHS with Dirichlet distribution is shown in Fig. 7, where $LDw_1$, $LDw_2$, and $LDw_3$ represent the sample values rectified by Dirichlet distribution for the three criteria. We can observe that the range of the sample values are recovered from 0 to 1, although there is

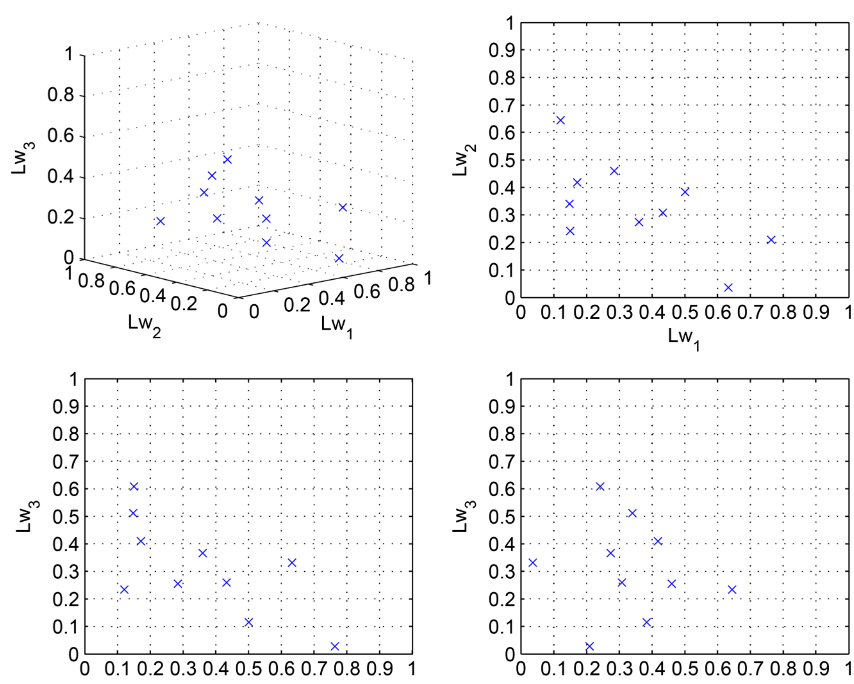


**Fig. 6 Normalized LHS in three dimensions and with two-dimensional projections.**

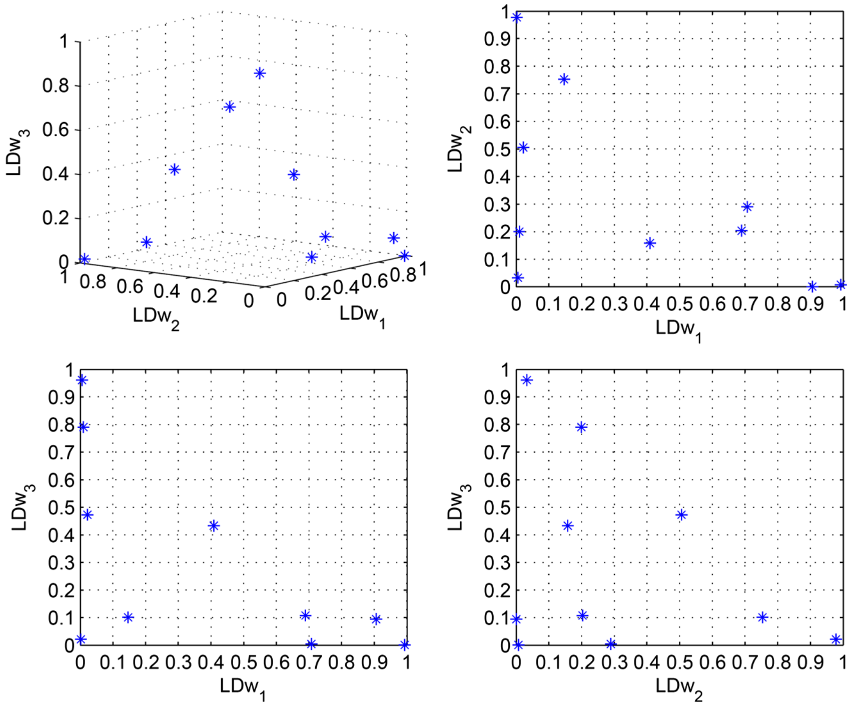


**Fig. 7 Modified LHS with Dirichlet distribution in three dimensions and with two-dimensional projections.**

not exactly one point in each row and each column in the two-dimensional projections. We should note that, when using the modified LHS with Dirichlet distribution, although the modified sample values are not strictly uniform any more, Dirichlet distribution can keep the ranges of the sample values larger once they are normalized, while maintaining the appealing Latin properties.

In this study, 100 sets of weighting factors are generated by the modified LHS with Dirichlet distribution. The weighing factors reflect the relative importance of the design criteria. For instance, a set of weighting factors $W_1 = [0.4333 \quad 0.0176 \quad 0.3719 \quad 0.1772]$ indicates that the first design criterion (OEM) is most important, followed by the third design criterion (utilization/block time) and the fourth design criterion (passenger density), while the second design criterion (fuel mass) is least important. The other 99 sets of weighting factors have similar explanations.

### B. Model Choice and Model Fitting

Response surface models have been widely used in surrogate model development in engineering design [35]. There are several advantages using response surface models, such as ease of implementation, minimal efforts required to train models, and ideality for uncertainty analysis. In this research, the response surface is used to construct the surrogate models. A widely used statistics software package JMP@ is employed to fit response surface models.

### C. Model Validation

The actual values versus the predicted values for the four design criteria are shown in Fig. 8. In the actual-by-predicted plot, the horizontal dotted line represents the mean of the actual values, the solid line shows the 45 deg diagonal line, and the two dotted lines

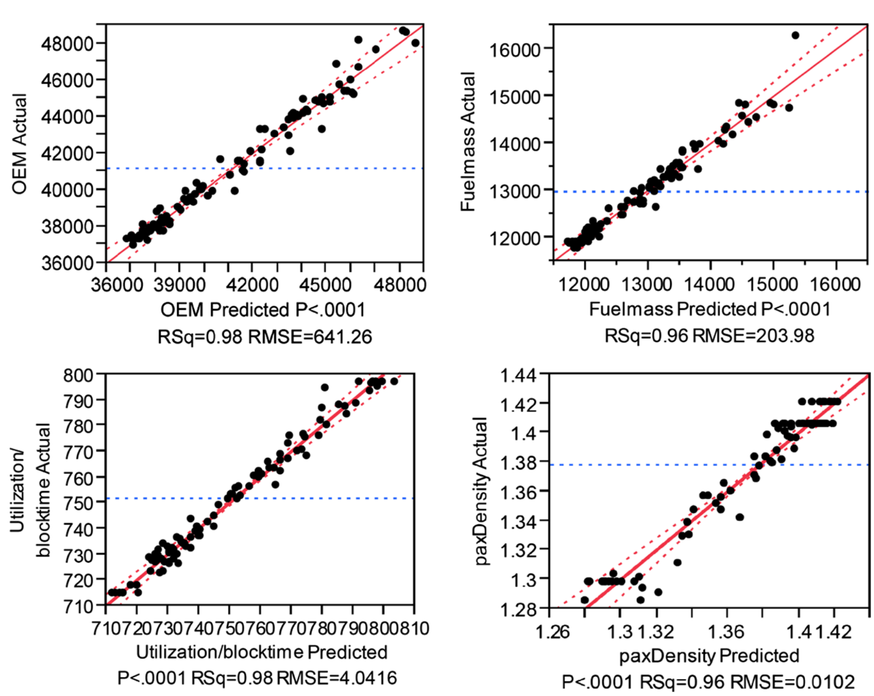


**Fig. 8 Actual by predicted plots of OEM, fuel mass, utilization/block time, and passenger density.**

Table 6 Diagnostics of response surface models for design criteria

| Diagnostics | OEM | Fuel mass | Utilization/block time | Passenger density |
|---|---|---|---|---|
| $R^2$ | 0.975 | 0.964 | 0.983 | 0.957 |
| $R^2_{\mathrm{Adj}}$ | 0.963 | 0.951 | 0.976 | 0.945 |
| RMSE, % | 1.56 | 1.57 | 0.54 | 0.74 |

along the diagonal show the 95% confidence intervals. The actual-by-predicted plots illustrate how well the predicted responses match the actual data. A quick assessment of the model is to eyeball a 45 deg pattern in these plots. In this example, the scatter plots all follow a 45 deg pattern along the diagonal line. This is one indicator of goodness of fit for the developed surrogate models.

The diagnostics of each response surface model, including $R^2$, $R^2_{\mathrm{Adj}}$, and rms error (RMSE) in percentage, are listed in Table 6. $R^2$ measures the proportion of the variation explained by the regressed polynomial model and ranges from 0 to 1; $R^2_{\mathrm{Adj}}$ adjusts the $R^2$ value to make it more comparable over models with different numbers of parameters; and RMSE estimates the standard deviation of the random error. The percent RMSE shown in Table 6 is normalized by its mean of response. The higher values of $R^2$ and $R^2_{\mathrm{Adj}}$ and lower values of percent RMSE are strong evidences of goodness of fit. Therefore, we can conclude that the developed surrogate models can provide adequate approximation to the analysis tool.

As discussed before, in the proposed multicriteria optimization framework, the computation time for one set of weighting factors is at least 5 min. A Monte Carlo-based uncertainty analysis with 1000 samples would take at least 84 h. With the developed surrogate models, the computation time depends on the number of weighting factors used for surrogate models development. In this example, the computation time for 100 sets of weighting factors is approximately 8 h. With the developed surrogate models, Monte Carlo based uncertainty analysis with 1000 samples takes only a few seconds. Thus, significant improvement has been made for efficient uncertainty analysis in aircraft conceptual design process.



## V. Conclusions

This paper explored the feasibility and added values of applying MCDA techniques in aircraft design. First, we established a new optimization framework incorporating MCDA techniques in aircraft conceptual design process. Then, we proposed an improved MCDA method (ITOPSIS) to aggregate the multiple design criteria into one composite figure of merit for the optimization. The ITOPSIS method is able to maintain ranking consistency for the top-ranked alternative. Furthermore, the assignment of weighting factors is one way to represent a designer's preferences in the design process. We solved the specification problem of weighting factors by using Latin hypercube sampling with Dirichlet distribution. Finally, we developed surrogate models for design criteria in terms of weighting factors to perform uncertainty assessment efficiently.

Although manufactures and airline communities focus on DOC as a prime optimization parameter, the objective of this paper is to explore the feasibility and to assess the added values of incorporating MCDA techniques in aircraft conceptual design process. A simple aircraft conceptual design example demonstrated that MCDA techniques can effectively help designers to specify their preferences and make tradeoffs among multiple performance criteria. The contribution of the paper lies in the cross-disciplinary research fields between aircraft design and multicriteria analysis.

In future work, to provide a more global optimization and include discrete design variables, hybrid optimizers combining genetic algorithms and gradient-based methods could be investigated for the proposed multicriteria optimization framework. For example, a discrete integer valued function for the optimization of passenger density is of future interest. The application of MCDA techniques could be extended to assess air transportation systems from the perspectives of multistakeholders, such as aircraft manufacturers, airlines, airports, air navigation service providers, and passengers, with the purpose of balancing social, economic, ecological, and technical constraints.

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## References

[1] Simpson, T., and Martins, J., "Multidisciplinary Design Optimization for Complex Engineered Systems: Report from a National Science Foundation Workshop," *Journal of Mechanical Design*, Vol. 133, No. 10, 2011, pp. 1–10. doi:10.1115/1.4004465

[2] Meller, F., "Key Buying Factors and Added Value—A New Approach to Aircraft Evaluation," *Proceedings of the DGLR Workshop — Aircraft Evaluation*, Technical Univ. of Munich, Garching, Germany, Oct. 1998, pp. 1–8.

[3] Dirks, G. A., and Meller, F., "Multidiciplinary Design Optimization—Enhanced Methodology for Aircraft and Technology Evaluation," *8th AIAA/USAF/ISSMO Symposium on Multidiciplinary Analysis and Optimization*, AIAA Paper 2000-4700, Sept. 2000.

[4] Stumpf, E., Langhans, S., Weiss, M., Sun, X., and Gollnick, V., "A Methodology for Holistic Air Transport System Analysis," *11th Aviation Technology Integration and Operations (ATIO) Conference*, AIAA Paper 2011-7010, Sept. 2011.

[5] Hwang, C. L., and Yoon, K., *Multiple Attribute Decision Making Methods and Applications: A State of the Art Survey*, Lecture Notes in Economics and Mathematical Systems, Springer–Verlag, Berlin, 1981, pp. 1–25.

[6] Ehrgott, M., Figueira, J., and Greco, S., *Trends in Multiple Criteria Decision Analysis*, Springer, New York/Dordrecht/Heidelberg/London, 2010, pp. 25–45.

[7] Saaty, T. L., *The Analytic Hierarchy Process*, Univ. of Pittsburgh, Pittsburgh, PA, 1988, pp. 5–26.

[8] Roy, B., "The Outranking Approach and the Foundations of ELECTRE Methods," *Theory and Decision*, Vol. 31, No. 1, 1991, pp. 49–73. doi:10.1007/BF00134132

[9] Brans, J., and Vincke, P., "A Preference Ranking Organization Method: The PROMETHEE Method for MCDM," *Management Science*, Vol. 31, No. 6, 1985, pp. 647–656. doi:10.1287/mnsc.31.6.647

[10] Yoon, K. P., and Hwang, C. L., *Multiple Attribute Decision Making: An Introduction*, SAGE Publ., Thousand Oaks, CA, 1992, pp. 9–40.

[11] Cvetkovic, D., "Evolutionary Multi-Objective Decision Support Systems for Conceptual Design," Ph.D. Thesis, Univ. of Plymouth, Plymouth, England, U.K., 2000.

[12] Marler, R., and Arora, J., "The Weighted Sum Method for Multi-Objective Optimization: New Insights," *Structural and Multidisciplinary Optimization*, Vol. 41, No. 6, 2010, pp. 853–862. doi:10.1007/s00158-009-0460-7

[13] Belton, V., and Gear, T., "On a Short Coming of Saaty's Method of Analytic Hierarchies," *Omega*, Vol. 11, No. 3, 1983, pp. 228–230. doi:10.1016/0305-0483(83)90047-6

[14] Keyser, W. D., and Peeters, P., "A Note on the Use of PROMETHEE Multicriteria Methods," *European Journal of Operational Research*, Vol. 89, No. 3, 1996, pp. 457–461.

[15] Chen, W., "On the Problem and Elimination of Rank Reversal in the Application of TOPSIS Method," *Operations Research and Management Science*, Vol. 14, No. 5, 2005, pp. 50–55.

[16] Wang, X., and Triantaphyllou, E., "Ranking Irregularities When Evaluating Alternatives by Using Some ELECTRE Methods," *Omega*, Vol. 36, No. 1, 2008, pp. 45–63. doi:10.1016/j.omega.2005.12.003

[17] Bandte, O., "A Probabilistic Multi-Criteria Decision Making Technique for Conceptual and Preliminary Aerospace System Design," Ph.D. Thesis, School of Aerospace Engineering, Georgia Inst. of Technology, Atlanta, 2000.

[18] Li, Y., Mavris, D., and DeLaurentis, D., "The Investigation of a Decision Making Technique Using the Loss Function," *AIAA 4th Aviation Technology, Integration and Operation (ATIO) Forum*, AIAA Paper 2004-6205, Sept. 2004.

[19] Kirby, M. R., "A Methodology for Technology Identification, Evaluation, and Selection in Conceptual and Preliminary Aircraft Desgin," Ph.D. Thesis, School of Aerospace Engineering, Georgia Inst. of Technology, Atlanta, 2001.

[20] Li, Y., "An Intelligent Knowledge-Based Multiple Criteria Decision Making Advisor for Systems Design," Ph.D. Thesis, Aerospace Systems Design Lab., School of Aerospace Engineering, Georgia Inst. of Technology, Atlanta, 2007.

[21] Lan, Y., Guan, Z., Jiao, Q., and Xu, G., "A Web-Based Computer-Aided Material-Selection System for Aircraft Design," *Journal of Computers*, Vol. 6, No. 5, 2011, pp. 976–983. doi:10.4304/jcp.6.5.976-983

[22] Messai, N., Melo, C., Hamdaoui, M., Bui, D., and Aufaure, M.-A., "Conceptual Analysis of Complex System Simulation Data for Decision Support: Application to Aircraft Cabin Design," *Proceedings of the 9th International Conference on Concept Lattices and Their Applications*, Fuengirola, Spain, 2012, pp. 199–210.

[23] Sullivan, K., Feigh, K., and German, B., "Impact of Visualization Tools on Design Decision Making," *12th AIAA Aviation Technology, Integration, and Operations (ATIO) Conference*, AIAA Paper 2012-5703, Sept. 2012.

[24] Tappeta, R. V., Messac, A., Renaud, J. E., and Sundararaj, G. J., "Interactive Physical Programming: Tradeoff Analysis and Decision Making in Multicriteria Optimization," *AIAA Journal*, Vol. 38, No. 5, 2000, pp. 917–926. doi:10.2514/2.1048

[25] Gurnani, A. P., and Lewis, K., "Using Bounded Rationality to Improve Decentralized Design," *AIAA Journal*, Vol. 46, No. 12, 2008, pp. 3049–3059. doi:10.2514/1.35776

[26] Hunt, B. J., Wiecek, M. M., and Fadel, G., "Matrices as Preference Modeling Tools in Bi-Criteria Engineering Design," *10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, AIAA Paper 2004-4320, Aug.–Sept. 2004.

[27] Huang, C.-H., and Bloebaum, C. L., "Incorporation of Preferences in Multi-Objective Concurrent Subspace Optimization for Multidisciplinary Design," *10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, AIAA Paper 2004-4548, Aug.–Sept. 2004.

[28] Barnum, G. J., and Mattsony, C. A., "A Computationally-Assisted Methodology for Preference-Guided Conceptual Design," *AIAA/ISSMO Multidisciplinary Analysis Optimization Conference*, AIAA Paper 2010-9175, Sept. 2010.

[29] Boehnke, D., Nagel, B., and Gollnick, V., "An Approach to Multi-Fidelity in Distributed Design Environments," *Proceedings of the IEEE Aerospace Conference*, IEEE Publ., Piscataway, NJ, 2011, pp. 1–10.

[30] Raymer, D., "Enhancing Aircraft Conceptual Design Using Multidisciplinary Optimization," Ph.D. Thesis, Royal Inst. of Technology, Stockholm, 2002.

[31] Heinze, W., "Methodical Design of Transport Aircraft I (Entwerfen von Verkehrsflugzeugen I)," Univ. of Brunswick—Institute of Technology, Brunswick, Germany, 2005.

[32] Jenkinson, L., Simpkin, P., and Rhodes, D., *Civil Jet Aircraft Design*, Arnold Publ., London, 1999, pp. 322–324.

[33] Lutsch, F., "Preliminary Aircraft Design Course," *Future Project*, Airbus SAS, Toulouse, France, 2008, pp. 13–14.

[34] Sun, X., and Li, Y., "An Intelligent Multi-Criteria Decision Support System for Systems Design," *10th Aviation Technology Integration and Operations (ATIO) Conference*, AIAA Paper 2010-9222, Sept. 2010.

[35] Forrester, A., Sobester, A., and Keane, A., *Engineering Design via Surrogate Modelling*, John Wiley & Sons, Ltd., Chichester, U.K., 2008, pp. 33–76.

[36] Lovison, A., and Rigoni, E., "Adative Sampling with a Lipschitz Criterion for Accurate Metamodeling," *Communications in Applied and Industrial Mathematics*, Vol. 1, No. 2, 2010, pp. 110–126.

[37] Mckay, M. D., Beckman, R. J., and Conover, W., "Comparison of 3 Methods for Selecting Values of Input Variables in the Analysis of Output From a Computer Code," *Technometrics*, Vol. 21, No. 2, 1979, pp. 239–245.