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Intelligent Multicriteria Decision Support System for Systems Design

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It is challenging to assess new technology in complex, interdisciplinary integrated systems, such as air transportation systems. Conflicting disciplines and technologies are always involved in aerospace systems design processes. Multicriteria Decision Analysis techniques can help decision makers to effectively deal with such situations and make wise design decisions. There is a variety of existing methods; thus, selection of the most appropriate method is critical because the use of inappropriate methods is often the cause of misleading design decisions. In this paper, an appropriateness index is used to quantify the goodness of a method for solving the problem under consideration. This method selection approach is implemented and an intelligent knowledge-based system is developed consisting of a multicriteria decision analysis library storing widely used decision analysis methods and a knowledge base providing the information required for the method selection process. Furthermore, a new approach for uncertainty assessment in the decision analysis process is proposed in this study. This novel approach for uncertainty assessment can be used to aggregate input data from tools with different fidelity levels and is capable of propagating uncertainties in an assessment chain. A business aircraft evaluation problem is conducted as a proof of implementation.

I. Introduction

It is challenging to assess new technology in complex, interdisciplinary integrated systems such as air transportation systems. There are large numbers of components with different characteristics in air transportation systems. The demands on air travel are increasing, not only regarding lower costs, but also better service quality, higher safety, and more environmental friendliness. The imperatives of air transport have evolved from "*higher, further, faster*" to "*more affordable, safer, cleaner, and quieter*" [1]. Vision 2020 set ambitious Advisory Council for Aeronautical Research in Europe goals for future air transportation systems in terms of quality and affordability, environment, efficiency, safety, and security [1]. To sustain the growth of air transport in the long term, multiple stakeholders in air transportation systems, such as manufacturers, airlines, and airports, are involved to meet these ambitious goals.

The focus of this research is one element in complex air transportation systems: aircraft. Aircraft are complex engineered systems involving multiple disciplines, such as aerodynamics, structures, and disciplines involving human behavior, which are extremely difficult to quantify and integrate into mathematical models and optimization problems [2]. Weckman et al. developed a decision support approach to model jet engine removal rates based on field data [3]. Scanlan et al. investigated cost modeling within the design process for a civil gas turbine engine [4]. However, the single economic criterion is not the only metric for technology evaluation as well as the figure of merit for design optimization. It is alerted that, by applying classic direct operating costs (DOC) comparison as the only yardstick in the evaluation of an aircraft, manufacturers run the risk of designing aircraft types and capabilities not fully suited to satisfy long-term transportation needs [5]. In addition to the economic consideration, there are several other criteria that need to be taken into account in aircraft design and evaluation processes, for instance, environmental impact and level of comfort. However, it is often difficult to derive a reliable transfer function to convert these nonmonetary values into monetary values [6]. One solution is to apply multicriteria decision analysis (MCDA) techniques.

As an important field in operational research, MCDA is a process that allows one to make decisions in the presence of multiple, potentially conflicting criteria [7–10]. Common elements in the decision analysis process are a set of design alternatives, multiple decision criteria, and preference information representing the attitude of a decision maker in favor of one criterion over another, usually in terms of weighting factors. MCDA techniques can help a decision maker to evaluate the overall performance of the design alternatives. Furthermore, MCDA techniques can aid in the generation, analysis, and optimization of design solutions.

MCDA techniques have been used to solve multicriteria decision problems in aircraft conceptual design and evaluation processes. Bandte developed a probabilistic MCDA method for multi-objective optimization and product selection [11]. However, it was pointed out that this method did not consider the absolute location of joint probability distribution [12]. Kirby applied the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method for the selection of technology alternatives in conceptual and preliminary aircraft design [13]. However, TOPSIS has limitations in that it assumes each criterion's utility is monotonic and it is rather sensitive to the weighting factors. Chen et al. evaluated four civil aircraft in terms of six criteria [14]: cost, performance, comfort, environmental influence, product support and family concept, and availability of aircraft. A 10-point ratio scale was employed to normalize the values

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of the six criteria, simple additive weighting (SAW) was used to rank the candidate aircraft. However, SAW is very sensitive to the normalization method and the weighting factors. Besides, Meller [5] and Meller and Dirks [15] assessed the civil aircraft by three criteria: DOC, operational commonality, and added values. The added values were quantified by equivalent DOC based on the weighting factors. However, inherent subjectivity and uncertainty of the weighting factors detriments the usefulness of this approach. Furthermore, Wang applied the TOPSIS method to evaluate seven initial training aircraft [16]. However, only technical criteria were considered because of the difficulty of collecting quantitative data.

From these applications of MCDA techniques in multicriteria decision problems in aircraft conceptual design and evaluation processes, two observations can be formulated:

Observation 1: There are various decision analysis methods that have been developed for solving multicriteria decision problems. Different methods have different underlying assumptions, analysis models, and decision rules that are designed for solving a certain class of decision-making problems. For example, SAW chooses the most preferred alternative that has the maximum weighted criteria values, whereas TOPSIS ranks the alternatives based on the Euclidean distance. This implies that it is critical to select the most appropriate method to solve a given problem because the use of inappropriate methods is often the cause of misleading design decisions. However, most researchers use one method without a formal method selection process; thus, the research area of decision analysis method selection has not drawn enough attention.

Observation 2: Because of different preferences and incomplete information, uncertainty always exists in the decision analysis process. When MCDA methods are used to solve decision problems, preference information describes the attitude of a decision maker in favor of one criterion over another. It is observed that the preference information is often highly uncertain considering that it is elicited based on a decision maker's experience or estimation. The inherent uncertainty has significant impacts on the final decision solution. This implies that it is critical to effectively capture and assess the uncertainty in the decision analysis process in order to get more accurate results.

As discussed before, currently there are various methods that have been developed to solve MCDA problems. Thus, it is necessary to review the existing methods, discuss in depth their advantages, disadvantages, applicability, computational complexity, etc. in order to choose the right method for the given problem for making better decisions. In this paper, 12 evaluation criteria are proposed to select the most suitable decision analysis method for the problem under consideration. Weighting factors are assigned to each evaluation criterion to describe a decision maker's preference information. An appropriateness index (AI) [17] is used to quantify the match between the decision analysis method and the problem under consideration. Furthermore, a new approach for uncertainty assessment in the decision analysis process is proposed in this study. The uncertainty assessment approach consists of three steps: uncertainty characterization, uncertainty analysis, and sensitivity analysis. This novel approach for uncertainty assessment can be used to aggregate input data from tools with different fidelity levels and is capable of propagating uncertainties in an assessment chain.

This paper is organized as follows. Section II reviews the background of decision analysis method selection. An advanced approach to effectively select the most appropriate decision analysis method for a given problem is formulated and presented in Sec. III. A new approach for uncertainty assessment in the decision analysis process is proposed in Sec. IV. To demonstrate the capabilities of the proposed decision support system, a business aircraft evaluation problem is conducted as a proof of implementation in Sec. V. Finally, conclusions are drawn for this study and presented in Sec. VI.

II. Background on Method Selection

Over the past decades, considerable research has been conducted to deal with the selection of the most appropriate method for a given decision problem. MacCrimmon first recognized the importance of method selection [18]. He proposed a taxonomy of decision analysis methods, created a method specification chart in the form of a tree diagram, and provided an illustrative application example [18]. Hwang and Yoon developed another tree diagram, which consists of nodes and branches connected by choice rules that can be used for selecting the decision analysis method for a specified problem [7]. Sen and Yang developed similar tree diagrams to help decision makers with selecting the appropriate methods, and the selection was based on the type of preference information elicited [19]. The tree diagram approach provides reasonable classification schemes and is easy to use. However, this approach has its own disadvantages: it usually gives two or more methods rather than the most appropriate method, and it only considers limited types of decision problems, preference information, and available methods. These limitations stop the tree diagram approach from being an effective solution to the method selection problem.

Possible criteria for evaluating decision analysis methods were proposed as an alternative solution to the method selection. Tecle developed an approach based upon a composite programming algorithm that aided in selecting an appropriate method [20]. They proposed four categories of criteria: decision maker-related characteristics, method-related characteristics, problem-related characteristics, and solution-related characteristics to evaluate a method. The independent criteria categories enable decision makers to conduct the evaluation in a specified order. However, it is difficult to quantify all decision analysis methods in terms of these four criteria categories. Besides, by using this approach, different users may get totally different results because the users' knowledge about the decision analysis methods has a strong impact on the final results.

Artificial intelligence techniques were employed by Poh [21] and Lu et al. [22] to help decision makers select the most suitable decision analysis method based on a series of user inputs. Poh [21] suggested a knowledge-based system, which allows decision makers to select the most appropriate method among available 11 multi-attribute decision-making methods. Lu et al. [22] proposed an intelligent system, which facilitated selecting the most suitable method among seven multi-objective decision-making methods. The knowledge-based intelligent system simplifies the method selection problem with simple questions by allowing direct selection or automated selection based on a decision maker's inputs. However, they do not clearly state the limitations or failure modes of the systems [23].

Hazelrigg suggested using the validation of decision analysis methods as a form of method evaluation and selection [24]. Ten desired properties were proposed to validate the methods. The author emphasized that a method is validate only if it is mathematically consistent and is derivable from an axiomatic basis. However, the validation of decision analysis methods is subjective and is dependent on the intended application of the method [23]. Roman et al. presented a conceptual framework for using decision analysis methods as attention directing tools, with reflection on the problem, method, and results interpretation [23]. However, the authors acknowledged that the conceptual framework is too generic to provide proper guidance for decision makers to use in engineering design.

Li developed a multicriteria interactive decision-making advisor for the selection of the most appropriate method [17]. However, only a limited number of methods was implemented and uncertainty propagation was not addressed explicitly. In this research, four major improvements are made in order to yield more accurate and reliable solutions [25]:

1) The distinction between filtering questions and scoring questions. The filtering questions are used to screen out inappropriate methods in the initial step of selection, and scoring questions are used to derive the attributes of a MCDA formulation.

2) The method library is extended to include all 16 widely used MCDA methods.

3) Two particular scenarios from the method implementation perspective are addressed: the case when there are two or more methods and the case when there is no method can be considered suitable for a given decision problem.

4) Most important, a new approach for uncertainty assessment in the decision analysis process is proposed and integrated into the intelligent multicriteria decision support system. This uncertainty assessment approach is discussed in detail in Sec. IV.

III. Advanced Approach for Method Selection

To effectively select the most appropriate decision analysis method for a given decision problem, an advanced approach is proposed in this study, as illustrated in Fig. 1. This approach consists of eight steps: define the problem, define evaluation criteria, perform initial screening, define preferences on evaluation criteria, calculate appropriateness index, evaluate decision analysis methods, choose the most suitable method, and conduct sensitivity analysis. Each step of the proposed approach to method selection is discussed in detail in the following subsections.

A. Step One: Define the Problem

The characteristics of the decision problem under consideration are addressed in the problem definition step, such as identifying the number of alternatives, attributes, and constraints. The available information about the decision problem is the basis on which the most appropriate decision analysis method will be selected and used to solve the problem.

B. Step Two: Define Evaluation Criteria

The proper determination of applicable evaluation criteria is important because they have great influence on the outcome of the method selection process. However, simply using every criterion in the selection process is not the best approach because the more criteria used, the more information is required, which will result in higher computational cost. Therefore, a tradeoff has to be made between the accuracy of the results and computational cost. In this study, the characteristics of decision analysis methods are identified by the relevant evaluation criteria in the form of a questionnaire. Twelve questions are defined to capture the advantages, disadvantages, applicability, and computational complexity of each decision analysis method.

1. *Filtering Questions*

1) Is the method able to handle selection or optimization problems?
2) Does the method allow tradeoffs among criteria?

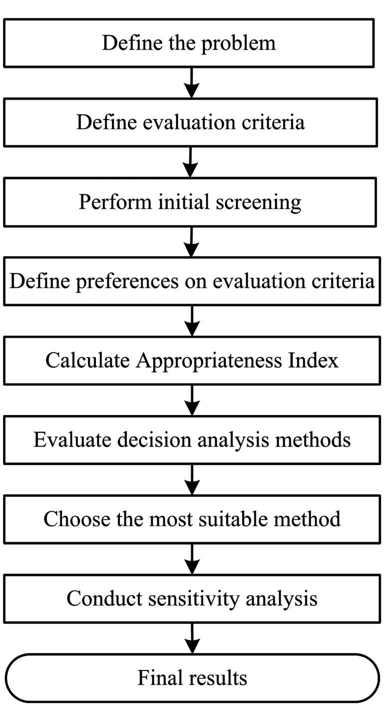

Fig. 1 An advanced approach for decision analysis method selection.

3) What input data are required by the method?

2. *Scoring Questions*

4) What preference information does the method need?
5) What decision rule does the method use to rank or sort the alternatives?
6) Does the method evaluate the feasibility of the alternatives?
7) Can the method handle any subjective attribute?
8) Does the method handle qualitative or quantitative data?
9) Does the method deal with discrete or continuous data?
10) Can the method handle the problem with hierarchy structure of attributes?
11) Is the method able to capture uncertainties existing in the problem?
12) Can the method support visual analytics?

It should be noted that the first three filtering questions are used to screen out inappropriate methods in the initial step of selection, whereas the other nine scoring questions are used to derive the information associated with the attributes of a MCDA formulation and the input data of decision matrix for method selection.

C. Step Three: Perform Initial Screening

In the initial screening step, filtering questions are used to screen out inappropriate methods. For the first filtering question, only scoring methods are suitable for solving optimization problems because the scores provided by the decision analysis methods can serve as objective functions in the optimization process, whereas classification methods, such as Elimination and Choice Translation Reality (ELECTRE), are not suitable because they cannot offer objective functions for optimization.

For the second filtering question, if tradeoffs among criteria are allowable, all noncompensatory methods will be removed, and only compensatory methods remain as candidate methods for further selection.

For the third filtering question, different decision analysis methods require different input; for example, most decision analysis methods require a decision matrix as input data, whereas Analytical Hierarchy Process (AHP) needs a pairwise comparison matrix [26]. When decision makers can provide a pairwise comparison matrix, AHP will be the only method left to solve the decision problem.

D. Step Four: Define Preferences on Evaluation Criteria

Usually, after the initial screening step is completed, more than one decision analysis method is expected to remain, otherwise we can directly choose the only one left to solve the decision problem. With the nine scoring questions defined in step 2, the decision maker's preference information on the evaluation criteria is defined. This will reflect which criterion is more important to decision makers in the method selection process.

In this study, weighting factors are assigned to each evaluation criterion to describe a decision maker's preference information. The weighting factors must be carefully elicited in order to accurately capture a decision maker's preferences. A subjective scale of 0 to 10 recommended by Hwang and Yoon [7] is used in this study, where 0 stands for extremely unimportant and 10 represents extremely important.

E. Step Five: Calculate Appropriateness Index

In this study, 16 widely used decision analysis methods are identified and stored in the knowledge base as candidate methods for selection. The evaluation criteria are captured by answering 12 questions relevant to the characteristics of the methods. An Appropriateness Index (AI) is used to rank the methods, given by Eq. (1) [17]:

$$AI_j = \frac{\sum_{i=1}^{n} w_i * b_{ji}}{\sum_{i=1}^{n} w_i * 1_i} * 100\%$$
$$b_{ji} = \begin{cases} 1 & \text{if } c_{ji} = a_i \\ 0 & \text{if } c_{ji} \neq a_i \end{cases} \quad i = 1, 2, \ldots, n; j = 1, 2, \ldots, m \tag{1}$$

where n is the number of evaluation criteria used to examine the methods with respect to the given problem, m is the number of methods stored in the knowledge base, $\{w_1, w_2, \ldots, w_n\}$ are the weighting factors for the evaluation criteria, a_i is the value of the ith characteristic of the decision problem, c_{ji} is the value of ith characteristic of the jth method, and b_{ji} is a Boolean number depending on the match of the ith characteristic of the decision problem and the ith characteristic of the jth method. If the ith characteristic of the decision problem matches the ith characteristic of the jth method, then $b_{ji} = 1$; otherwise, $b_{ji} = 0$. 1_i denotes one.

For one set of weighting factors, the numerator of AI ($\sum_{i=1}^{n} w_i * b_{ji}$) calculates the weighted score for each method, whereas the denominator ($\sum_{i=1}^{n} w_i * 1_i$) calculates the maximum value if the characteristics of one method match completely with the characteristics of the decision problem. For each method, AI is calculated by the weighted score normalized by the maximum value. AI ranges from 0 to 100%. Thus, higher value of AI indicate the method is more appropriate to solve the given decision problem.

Table 1 shows one example of the AI calculation process for the TOPSIS method. At first, decision makers identify key characteristics of the decision problem and define weighting factors for evaluation criteria. In this example, decision rule, input data, and uncertainty analysis are considered as the most important criteria, and so high weights are assigned to these evaluation criteria. The other evaluation criteria are assigned in the same way. The weighting factors for the nine evaluation criteria are defined as $[5\ 8\ 4\ 4\ 6\ 4\ 3\ 6\ 5]$. Second, the characteristics of the decision problem are obtained from the answers to the questionnaire, whereas the characteristics of the decision analysis methods can be obtained from the knowledge base, where the characteristics of the methods are predefined. Then, the characteristics of the problem and method are compared pairwise in order to see if they match with each other. Finally, AI can be calculated for TOPSIS by using Eq (1), and the result is given by Eq. (2).

$$AI_{\text{TOPSIS}} = \frac{\sum_{i=1}^{9} w_i * b_{ji}}{\sum_{i=1}^{9} w_i * 1_i} * 100\%$$
$$= \frac{[5\ 8\ 4\ 4\ 6\ 4\ 3\ 6\ 5] * [1\ 1\ 0\ 1\ 1\ 1\ 1\ 0\ 1]^T}{[5\ 8\ 4\ 4\ 6\ 4\ 3\ 6\ 5] * [1\ 1\ 0\ 1\ 1\ 1\ 1\ 0\ 1]^T}$$
$$* 100\% = \frac{35}{45} * 100\% = 78\% \tag{2}$$

F. Step Six: Evaluate Decision Analysis Methods

To compare the appropriateness of the methods with respect to the given decision problem, each method is evaluated based on nine scoring questions and their AIs are obtained. Based on the AI calculation, the method with the highest score will be chosen as the most appropriate one to solve the original decision problem.

G. Step Seven: Choose the Most Suitable Method

As noted in step 6, the method with the highest AI will be recommended as the most appropriate method to solve the given problem. The developed decision support system is used to guide the user to reach the final decision when solving the decision problems. After one decision analysis method is identified as the most appropriate method, the user can simply click the name of the method, and the methodology instructions will be displayed to guide the user to solve the given problem. The mathematical calculation steps are built in the MATLAB-based decision support system; thus, the user can just simply follow the instructions, such as inputting necessary data, to get the final result.

H. Step Eight: Conduct Sensitivity Analysis

It is observed that different decision makers often have different preference information on the nine scoring questions; thus, sensitivity analysis should be performed on the method selection algorithm to analyze its robustness with respect to the variations of the weighting factors.

To accommodate different preference information from different decision makers, weighting factor of each characteristic is treated in a parametric manner. In our integrated user interface, decision makers can adjust criteria weights by moving the corresponding slide bars. It is worth noting that there is no absolute best decision analysis method that can solve any decision problem because the method selection is problem specific. The selection of the most suitable decision analysis method depends on the problem under consideration.

I. Two Particular Scenarios During the Method Selection Process

There are two scenarios of particular interest that need to be considered during the method selection process: 1) the case when there are two or more methods whose appropriateness scores are the highest, and 2) the case when there is no method that can be considered suitable for a given problem. These two particular scenarios were not addressed in the previous research in [17]. In this study, these two particular scenarios are explicitly addressed and formulated as follows.

For the first scenario, when there are more methods that can be considered as the best ones to solve a given decision problem, the decision maker can perform uncertainty analysis of the weighting factors for the nine evaluation criteria. The method that has the highest probability to be ranked first is recommended as the most suitable method for the decision problem under consideration. In the developed multicriteria decision support system, the decision maker can adjust the weighting factors of the nine evaluation criteria by moving the corresponding slide bars.

For the second scenario, when there is no method can be considered as the suitable one for a given decision problem, new methods or hybrid methods need to be used to solve the given problem. During the process of method selection, more insights on the characteristics of the methods can be obtained. For example, by

Table 1 Appropriateness index calculation process for TOPSIS

	Criteria weights	Problem criteria values	Method criteria values	Match scores
Evaluation criteria	w_i	a_i	c_{ji}	b_{ji}
		Filtering questions		
1) Selection/optimization	–	–	–	–
2) Allow tradeoff	–	–	–	–
3) Input data	–	–	–	–
		Scoring questions		
4) Preference information	5	Relative weight	Relative weight	1
5) Decision rule	8	Minimum closeness	Minimum closeness	1
6) Feasibility evaluation	4	Yes	No	0
7) Subjective	4	No	No	1
8) Qualitative/quantitative data	6	Quantitative	Quantitative	1
9) Discrete/continuous data	4	Discrete	Discrete	1
10) Single/hierarchy	3	Single	Single	1
11) Capture uncertainties	6	Yes	No	0
12) Visualization	5	Yes	Yes	1

combining two or more decision analysis methods, decision makers may get one hybrid method that is more effective for solving the given problem. Moreover, the definition of a threshold value for the appropriateness index of the decision analysis method can be helpful to identify the occurrence of the second scenario. This is a future work that needs further investigation in the method selection process.

IV. Uncertainty Assessment in the Decision Analysis Process

As discussed in Sec. I, inherent uncertainties associated with the input data have significant impacts on the final decision for a decision problem. Considerable research has been conducted to assess the uncertainties propagated in the decision analysis process. Durbach and Stewart provided a review of uncertainty modeling for conducting multicriteria decision analysis with uncertain attribute evaluations [27]. The review included models using probabilities, quantiles, variances, fuzzy numbers, and scenarios. Aschough et al. discussed the incorporation of uncertainty in environmental decision-making processes [28]. Especially, the authors asserted the importance of developing innovative methods for quantifying the uncertainty associated with human input.

Many researchers used simulation-based techniques to solve uncertain multicriteria decision problems [29–31]. Hyde et al. [32] and Hyde and Maier [33] developed an uncertainty analysis program in Excel for two decision analysis methods. Allaire and Willcox proposed a surrogate modeling method for propagating uncertainty from model inputs to model outputs [34]. However, these uncertainties were directly defined in terms of probability distributions and decision makers' confidence levels regarding these uncertainties were not explicitly captured. Consequently, the quality of the final decision made under these uncertainties cannot be guaranteed.

In this research, a novel approach for uncertainty assessment in the decision analysis process is proposed. This approach consists of three steps: uncertainty characterization by percentage uncertainty with confidence level, uncertainty analysis using error propagation techniques, and sensitivity analysis based on iterative binary search algorithm. Each step of the uncertainty assessment approach is discussed in detail in the following sections.

A. Uncertainty Characterization by Percentage Uncertainty with Confidence Level

The uncertainty characterized by percentage uncertainty with confidence level is converted into standard deviation through the inverse error function. For a normal random variable X with $N(\mu, \sigma^2)$ distribution, the probability of a random sample value falling within the interval $[\mu - n\sigma, \mu + n\sigma]$ can be calculated by Eq. (3):

$$P(\mu - n\sigma < X\mu + n\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \int_{\mu-n\sigma}^{\mu+n\sigma} e^{(-(x-\mu)2/2\sigma^2)\,\mathrm{d}x} \tag{3}$$

The error function is shown in Eq. (4) [35]. With the substitution $z = \frac{X-\mu}{\sigma}$, Eq. (3) can be converted into Eq. (5):

$$y = erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{(-t^2)}\,\mathrm{d}t \tag{4}$$

$$P(\mu - n\sigma < X < \mu + n\sigma) = \frac{1}{\sqrt{2\pi}} \int_{-n}^{n} e^{[-(z^2/2)]}\,\mathrm{d}z = erf\left(\frac{n}{\sqrt{2}}\right) \tag{5}$$

When the probability (confidence level) of a normal random variable X falling within a certain confidence interval is given, the numbers of standard deviation can be calculated by the inverse error function, as shown in Eq. (6):

$$n = \sqrt{2} erf^{-1}(\text{Confidence level}) \tag{6}$$

Note that the relative error here is equal with the percentage uncertainty; thus, the conversion of percentage uncertainty into standard deviation is shown in Eq. (7):

$$\sigma = \frac{\text{Percentage uncertainty}(\%)\mu}{n} \tag{7}$$

This novel approach for uncertainty characterization is capable of propagating uncertainties in an assessment chain, where input data from various tools with different fidelity levels need to be aggregated. In this case, the fidelity level can be represented by the confidence level and then be converted into the number of standard deviation using Eq. (6).

B. Uncertainty Analysis Using Error Propagation Techniques

Error propagation techniques answer the question: How the uncertainties of input variables will be propagated to some predefined functions involving these variables and lead to the final result [36]? There are two classes of error propagation techniques: analytical and simulation-based error propagation techniques. The analytical error propagation technique relies on a linearized Taylor series expansion of the function about the mean of each variable. The total error of the function is obtained by combining the linearized individual error in quadrature [36]. Although analytical error propagation technique is appropriate for simple calculation processes, simulation-based error propagation technique is more suitable for dealing with complex models, where a tradeoff has to be made between result accuracy and computation time. In this study, the uncertainty analysis using simulation-based error propagation techniques is illustrated in Fig. 2.

C. Sensitivity Analysis via Iterative Binary Search Algorithm

Sensitivity analysis addresses the question how the variation of input variables influences model output [37]. In this study, an iterative binary search algorithm is developed to investigate the sensitivity of alternatives' ranking to the variations of input data. The binary search technique has been widely used to find a target value in a sorted (usually ascending) sequence efficiently [38,39]. This technique compares the middle element of the sorted sequence to the target value. If the middle element is equal to the target value, then the search terminates. If the target value is less than middle element, then the algorithm eliminates the right half of the sorted sequence and conducts the same search for the left side. If the target value is bigger than the middle element, then the algorithm ignores the left half of the sorted sequence and performs the same search for the right side. Otherwise, we can conclude that the target value is not in the sorted sequence. For example, given a sorted sequence $[0 \quad 5 \quad 12 \quad 17 \quad 23 \quad 25 \quad 50 \quad 60 \quad 80]$, assume that we want to find the target value 25. The binary search technique works as follows.

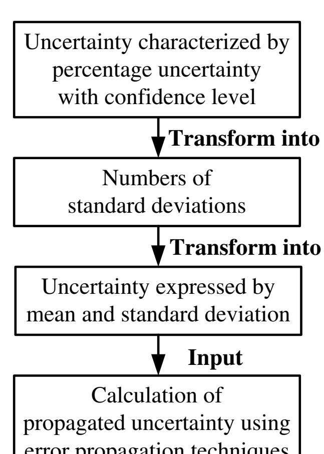

Fig. 2 The process of uncertainty analysis using error propagation techniques.

1) First iteration: $[0 \quad 5 \quad 12 \quad 17 \quad 23 \quad 25 \quad 50 \quad 60 \quad 80]$. The target value 25 is bigger than the middle element 23. Ignore the left half of the sorted sequence and perform the same search for the right side.

2) Second iteration: $[25 \quad 50 \quad 60 \quad 80]$. The target value 25 is smaller than the middle element 50. Ignore the right side of the sorted sequence and perform the same search for the left side.

3) Third iteration: [25]. The target value 25 equals the element 25. The target value is found.

When using MCDA methods to solve a given decision problem, main input data are the values of decision criteria and a decision maker's preference information. The iterative binary search algorithm varies one input variable at a time in order to find the minimum change that can alter the ranking of two alternatives. The sensitivity analysis can provide good insights about the range of an input variable when the ranking of the alternatives preserves. It can also tell a decision maker which input variable is more sensitive to the ranking of the alternatives and more attention needs to be paid to the value of this input variable.

V. Implementation

In this section, a business aircraft evaluation problem is conducted to demonstrate the capabilities of the proposed multicriteria decision support system. A three-step framework is implemented: definition of a decision making problem, selection of the most appropriate MCDA method for the given problem, and uncertainty assessment in the decision analysis process. This three-step framework provides a general guideline on how to structure and solve any given decision-making problem. In this implementation, emphasis is put on explaining the holistic process of the intelligent multicriteria decision support system. Thus, the step-by-step problem-solving process is explained and discussed for this decision problem.

A. Selection of the Most Appropriate MCDA Method

1. *Step One: Define the Problem*

With increasing demand on air travel, business aircraft are popular alternatives for wealthy aviation. At present, there are six major business jet manufacturers: Canadian Bombardier, American Cessna, French Dassault, Brazilian Embraer, American Gulfstream, and American Hawker. There are more than 40 different types of business aircraft available in the current market, costing from 1 million to almost 100 million [40]. How to choose the appropriate aircraft to meet the needs of business aviation customers is a complicated multicriteria decision process.

Traditional evaluation is dominated by economic criteria, such as purchase price and operating costs. However, the success of an aircraft is no longer dominated by these economic criteria [15]. In addition to costs, there are several other criteria that need to be evaluated at the same time, for instance, aircraft performance, environmental impacts, and level of comfort. Therefore, considering these multiple conflicting criteria simultaneously, the evaluation and selection of a business jet is a typical multicriteria decision problem and needs to be prudently conducted.

In this study, for the business aircraft evaluation problem, the selection of the most appropriate decision analysis method is conducted first using the proposed multicriteria decision support system, as described in Sec. III. Then, this most suitable method is used to evaluate the business aircraft for business aviation customer. Ten evaluation criteria are identified for the business aircraft evaluation problem, including seven hard technical criteria (typical passenger seat number, maximum range, purchase price, fuel consumption per seat kilometer, high-speed cruise speed, takeoff field length, and noise) and three quantified soft criteria (cabin volume per passenger, product support level, and manufacturer's reputation).

Empirical studies in consumer behavior and industrial market context have shown that the quality of a decision has an inverted U-shaped relationship with the number of alternatives, and the number of intensively discussed alternatives is less than five [41]. In practice, a small number of alternatives can be obtained by a simple checklist of desirable features [42]. In this study, the use of filter criteria can highly facilitate evaluating the business aircraft by reducing the number of alternatives under consideration.

In the business aircraft evaluation problem, typical passenger seat number, maximum range, and purchase price are used as filter criteria for initial screening in the first phase of the decision process. Fuel consumption per seat kilometer, high-speed cruise speed, takeoff field length, noise, cabin volume per passenger, product support level, and manufacturer's reputation are identified as seven decision criteria for the business aircraft evaluation problem.

Assume that one business aviation customer considers to purchase a business jet with eight to 10 typical passengers on board. Aircraft range with maximum fuel and available payload should be around 5500 to 6500 km, and purchase price is between 20 and 25 million. In the available business jet market, four business jet alternatives satisfy the needs of the customer. The values of three filter criteria and seven decision criteria for the four business jet alternatives are summarized in Table 2.

2. *Step Two: Define Evaluation Criteria*

To identify the most appropriate method, a method base needs to be built from where the most appropriate method can be selected. In this study, 16 widely used decision analysis methods are studied and their characteristics are stored in the knowledge base. To compare the appropriateness of the methods with respect to the given problem, each method is assessed based on the proposed 12 evaluation criteria. The 12 evaluation criteria are captured by answering 12 questions, as shown in Fig. 3.

3. *Step Three: Perform Initial Screening*

To get the most appropriate method, infeasible decision analysis methods are eliminated by three filtering questions. For the business aircraft evaluation problem, with the assumption that tradeoffs among criteria are not permitted, all compensatory methods are excluded and only noncompensatory methods remain as candidate methods for further selection.

4. *Step Four: Define Preferences on Evaluation Criteria*

Because decision makers may consider one criterion as more important than another when selecting the most appropriate method, a weighting factor is defined for each criterion to reflect a decision maker's preference information. The decision maker's preference information on the evaluation criteria can be defined using slide bars in the integrated user interface, where 0 stands for an extremely unimportant criterion and 10 represents an extremely important criterion.

5. *Step Five: Calculate Appropriateness Index*

In this step, an appropriateness index (AI) for each decision analysis method is calculated by Eq. (1). Essentially, AI is used to determine how the characteristics of a method match the characteristics of the given problem.

6. *Step Six: Evaluate Decision Analysis Methods*

According to step 5, AI of decision analysis methods are obtained and presented in Fig. 4, where a higher score represents greater appropriateness of the method for the given problem.

7. *Step Seven: Choose the Most Suitable Method*

In this example, as indicated in Fig. 4, ELECTRE I gets the highest score among the decision analysis methods; therefore, it is selected as the most appropriate method to solve the business aircraft evaluation problem. Its mathematical calculation steps are built in the decision support system; thus, decision makers can simply click the name of the method, and methodology instructions of ELECTRE I will be displayed to guide decision makers to solve the given problem and get the final solution.

8. *Step Eight: Conduct Sensitivity Analysis*

The preference to the nine scoring questions can be varied in the method selection process. In our integrated user interface, decision

Table 2 Values of evaluation criteria for four business jet alternatives

	Alternatives			
	A_1 Bombardier Challenger 300	A_2 Cessna Citation X	A_3 Gulfstream G200	A_4 Hawker H4000
				
Filter criteria				
F_1: Typical passenger seat number	8	9	10	8
F_2: Maximum range, km	5975	5656	6378	5808
F_3: Purchase price, millions of dollars	24.7500	21.6330	23.3250	22.9089
Decision criteria				
C_1: Fuel consumption per seat kilometer, kg/pax/km	0.2396	0.2720	0.2264	0.2624
C_2: High-speed cruise speed, km/h	870	952	870	870
C_3: Takeoff field length, m	1466	1567	1854	1545
C_4: Noise, EPNdB	84.2333	82.4333	86.7333	86.1000
C_5: Cabin volume per passenger, m^3/pax	4.0500	2.3556	3.1000	3.4375
C_6: Product support level	7.63	8.22	7.75	7.66
C_7: Manufacturer's reputation	55	39	82	78

Note: "pax" stands for "passenger".

makers can adjust the weighting factor of each criterion by moving the slide bars. In this example, with the current input data, it is observed from Fig. 4 that ELECTRE I is ranked first by the multicriteria decision support system. Therefore, ELECTRE I is further used to solve the business aircraft evaluation problem.

B. Evaluation Results Using ELECTRE I

ELECTRE methods use the concept of outranking relation [43,44]. An alternative is dominated if there is another alternative that excels it in one or more criteria and equals it in the remainder. A nondominated alternative is one in which no criteria can be improved without a simultaneous detriment to at least one of the others.

When ELECTRE I is used to solve the business aircraft evaluation problem, it requires a decision matrix as input data and weighting factors as the representation of a decision maker's preference information. For this example, the decision matrix is shown in matrix D, where each row corresponds to one business jet alternative, and each column corresponds to one decision criterion. In the first round of evaluation, equal weighting factors are considered, as shown in vector W. The evaluation results using ELECTRE I are shown in matrix $M_{\text{aggregated dominance}}$:

$$D = \begin{bmatrix} 0.2396 & 870 & 1466 & 84.2333 & 4.0500 & 7.63 & 55 \\ 0.2720 & 952 & 1567 & 82.4333 & 2.3556 & 8.22 & 39 \\ 0.2264 & 870 & 1854 & 86.7333 & 3.1000 & 7.75 & 82 \\ 0.2624 & 870 & 1545 & 86.1000 & 3.4375 & 7.66 & 78 \end{bmatrix}$$

$$W = [0.1429 \;\; 0.1429 \;\; 0.1429 \;\; 0.1429 \;\; 0.1429 \;\; 0.1429 \;\; 0.1429]^T$$

$$M_{aggregated\ dominance} = \begin{array}{cc} & \text{is dominated by} \\ \text{dominates} & \begin{bmatrix} - & 1 & 0 & 0 \\ 0 & - & 0 & 0 \\ 1 & 0 & - & 0 \\ 0 & 1 & 0 & - \end{bmatrix} \end{array}$$

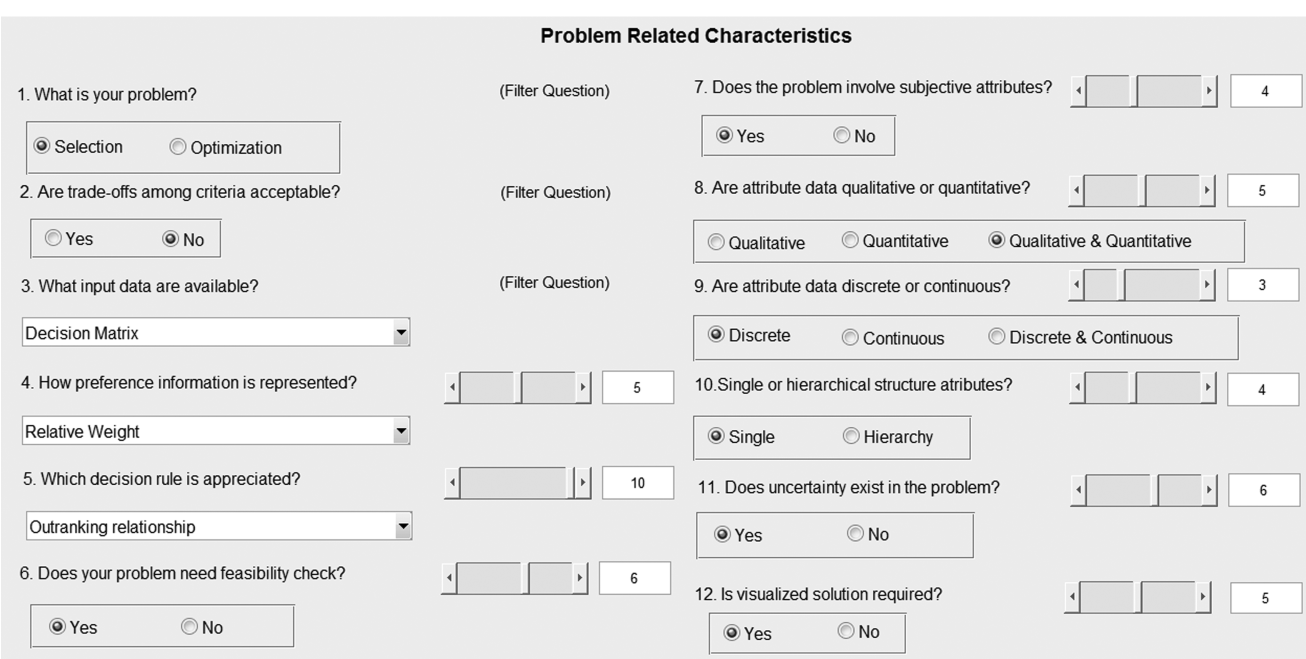

Fig. 3 Questions related to evaluation criteria for method selection in business aircraft evaluation process.

In the aggregated dominance matrix, element 1 in each column indicates that this alternative is dominated by other alternatives. In this example, A_1 and A_2 are dominated by A_3 and A_4. Therefore, we can obtain that, when the weighting factors are evenly distributed among the seven criteria, A_1 (Bombardier Challenger 300) and A_2 (Cessna Citation X) should be excluded from the candidates of business jets.

However, the outranking relationship between A_3 (Gulfstream G200) and A_4 (Hawker H4000) cannot be identified with one set of equal weighting factors. Uncertainty assessment for the weighting factors is further investigated in the following section.

C. Uncertainty Assessment for ELECTRE I

In the business aircraft evaluation problem, weighting factors are used to represent a decision maker's preference information. To identify the outranking relationship between A_3 and A_4, it is critical to effectively capture the uncertainty associated with the preference information and assess its impact on the final decision solution. In this subsection, uncertainty assessment for the weighting factors is performed, following the new uncertainty assessment approach proposed in Sec. IV.

1. Uncertainty Characterization

Uncertainties for the weighting factors are represented by percentage uncertainties with confidence levels. For example, if a decision maker assigns 15% uncertainty to the weighting factor of the first decision criterion w_1 with 90% confidence level, it implies that the decision maker is 90% confident that w_1 would fall within the interval $[w_1(1-15\%), w_1(1+15\%)]$. For this example, the uncertainty characterization for the weighting factors is summarized in Table 3.

Percentage uncertainties with confidence levels are transferred into standard deviations using Eqs. (6) and (7). When the weighting factors are evenly distributed among the seven decision criteria, the mean of weighting factors μ_W is equal to normalized weighting factors, and the standard deviation of weighting factors is σ_W, denoted as follows:

$$\mu_W = [0.1429\ \ 0.1429\ \ 0.1429\ \ 0.1429\ \ 0.1429\ \ 0.1429\ \ 0.1429]^T$$

$$\sigma_W = [0.0130\ \ 0.0073\ \ 0.0149\ \ 0087\ \ 0.0345\ \ 0.0335\ \ 0.0261]^T$$

For instance, the standard deviation of w_1 with 15% uncertainty at 90% confidence level is calculated by Eqs. (8) and (9), respectively:

$$n_{w_1} = \sqrt{2}erf^{-1}(\text{Confidence level}) = \sqrt{2}erf^{-1}(90\%) = 1.6449 \tag{8}$$

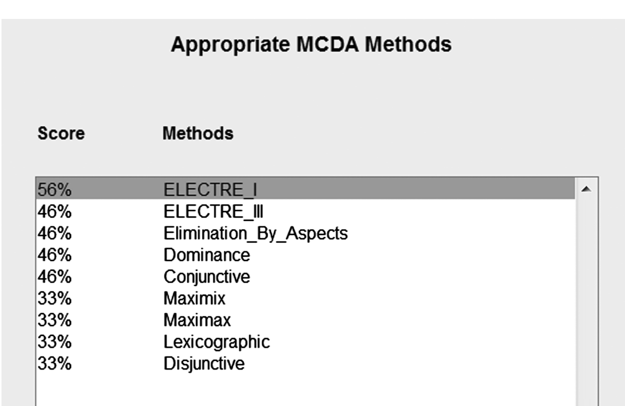

Fig. 4 Decision analysis methods ranking list in business aircraft evaluation process.

Table 3 Uncertainty characterization for weighting factors

	Weighting factors						
	w_1	w_2	w_3	w_4	w_5	w_6	w_7
Percentage uncertainty, %	15	10	15	10	25	30	30
Confidence level, %	90	95	85	90	70	80	90

Table 4 Probabilistic outranking relationships

	Alternatives			
	A_1	A_2	A_3	A_4
Nondominated, %	48.84	11.50	89.22	99.71
Dominated, %	51.16	88.50	10.78	0.29

$$\sigma_{w_1} = \frac{\text{Relative error}(\%)\mu_{w_1}}{n_{w_1}} = \frac{(15\%)(0.1429)}{1.6449} = 0.0130 \tag{9}$$

In this step, uncertainties in the weighting factors are transferred into means and standard deviations. The variables μ_W and σ_W are the input for the error propagation calculation in the uncertainty analysis step.

2. Uncertainty Analysis

Monte Carlo-based numerical error propagation technique is applied to perform uncertainty analysis for ELECTRE I. Ten thousand runs are performed from normal distribution with parameters μ_W and σ_W. The probabilistic outranking relationships for each alternative are presented in Table 4. It can be observed that, with evenly distributed weighting factors among the seven decision criteria, A_4 (Hawker H4000) has the highest probability to be nondominated, whereas A_2 (Cessna Citation X) has the highest probability to be dominated.

Besides, it is noted that the probability of A_1 to be a nondominated alternative or dominated alternative is approximately equal. To investigate the unstable status of A_1, sensitivity analysis for the alternatives to the weighting factors is further conducted in the following section.

3. Sensitivity Analysis

In the business aircraft evaluation problem using ELECTRE I, with equally distributed weighting factors among seven criteria, A_1 and A_2 are dominated alternatives, whereas A_3 and A_4 are nondominated alternatives. Based on the iterative binary search algorithm developed in Sec. IV, the minimum changes in the weighting factors that can alter the nondominance or dominance status of alternatives are summarized in Table 5, where N/F (Nonfeasible) means that it is not mathematically feasible to alter the nondominance or dominance status of alternatives through the change of the current parameter.

It can be seen from the first row in Table 5 that it is not feasible to change the weighting factor of C_2 to switch A_1 into a nondominated alternative, whereas only around 3% change in the weighting factors of C_5 or C_7 can make A_1 become a nondominated alternative. Therefore, we can conclude that A_1 (Bombardier Challenger 300) is most sensitive to the weighting factor of C_5 (cabin volume per passenger) and the weighting factor of C_7 (manufacturer's reputation). The sensitivity of A_1 explains well its unstable status observed from Table 4 in the preceding section.

4. Uncertainty Assessment Insights for the Business Aviation Customer

The uncertainty assessment conducted here helps the business aviation customer to prioritize the evaluation information when selecting a business jet. For the scenario considered in this study, the business aviation customer should pay more attention to the business jet A_1 (Bombardier Challenger 300) because it is most sensitive to the

Table 5 Minimum changes in the weighting factors to alter the nondominance or dominance status of alternatives

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1 to nondominance, %	−27.67	N/F	29.05	50.06	3.33	−49.99	−3.41
A_2 to nondominance, %	−49.99	33.39	−49.99	50.06	−49.99	50.06	−49.99
A_3 to dominance, %	−18.98	406.80	22.65	1010.50	89.59	1186.95	−44.20
A_4 to dominance, %	60.69	616.37	−38.47	1349.15	177.39	837.45	−58.81

preference information on the evaluation criteria. Furthermore, the business aviation customer should consider carefully cabin volume per passenger and manufacturer's reputation because the preference information for the two evaluation criteria is rather sensitive to the best business jet alternative.

VI. Conclusions

In this study, a systematic decision analysis method selection process is developed and applied to solve a given decision problem. The selection of the most appropriate decision analysis method is formulated as a complicated MCDA problem and an advanced approach is proposed to solve this problem. The method evaluation criteria for selecting the most appropriate method are defined. Weighting factors are assigned to each evaluation criterion to describe a decision maker's preference information. An appropriateness index is used to quantify the match between the method and the problem under consideration.

Furthermore, a new uncertainty assessment approach in the decision analysis process is proposed, consisting of uncertainty characterization, uncertainty analysis, and sensitivity analysis. This approach can be used to verify the final decision for a given problem when the uncertainties of input data are introduced. This novel approach for uncertainty assessment can be used to aggregate input data from tools with different fidelity levels and is capable of propagating uncertainties in an assessment chain. A business aircraft evaluation problem is implemented to demonstrate the capabilities of the intelligent multicriteria decision support system. Our study shows that the proposed decision support system can effectively help decision makers with selecting the most appropriate method and guiding decision makers to get the final decision for the given decision problem.

In addition, a three-step framework for solving decision making problems is proposed and implemented in this research: definition of a decision making problem, selection of the most appropriate MCDA method for the given problem, and uncertainty assessment in the decision analysis process. This three-step framework provides a general guideline on how to structure and solve any given decision making problem.

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