Stochastic Tail Assignment Problem under Disruption Recovery Initiative

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***Abstract*—The tail assignment problem(TAP), which assigns flight sequences to specific aircraft, serves as an indispensable part of airline planning process in terms of safe and efficient operation. As the airline industry is always faced with numerous schedule disruptions, there has been a growing focus on taking uncertainty into account during schedule making. This paper presents a new stochastic model for TAP in order to provide robust flight schedules when confronted with operational perturbations (e.g. flight delays, airport disclosure). The model is formulated as integration of a master problem and a recourse subproblem under the framework of stochastic programming. The corresponding solution algorithm involves improved column generation and Benders decomposition with an objective to minimize both operational cost and expected recovery cost under a bunch of disruption scenarios. To demonstrate the benefits of stochastic TAP model, a computational study based on real airline data is performed to assess the performance of deterministic and stochastic TAP model respectively.**

***Index Terms*—Airline planning, Robustness, stochastic programming**

I. Introduction

The rapid growth of the air transportation has triggered great pressure on civil aviation stakeholders and infrastructure. This complex system has interactions with several its components that make delay or other disruption inevitable [1]. As a consequence, even minor disruptions are likely to cause cascaded effects on operation efficacy. The year 2018 in US witnessed a situation that around 18.77% of all arrival flights were delayed by more than 15 minutes while 1.72% of that were cancelled. [2] With a predicted growth of air traffic, such irregular perturbations may exert more impact on air industry.

As one of the important component of aviation industry, airline sector has always been a leader in developing and applying advanced optimization techniques to cope with complex planning process. While the airline process is usually carried out under assumptions that operation will be executed as originally planned. The aforementioned irregular delays or cancellations, more often than not, disturb schedules and affect on time performance due to various reasons like convective weather, propagated delay. Thus, the original schedule is susceptible to disruption and infeasibility considering various operational constraints(e.g. maintenance requirements). The considerable cost in airline irregularity has spurred great interests in robust airline planning among industry and academia.

To mitigate the discrepancy between scheduled plan and actual operation, the robust approaches in research can be classified as three main categories: (1) domain-specific approaches, (2) propensity-diminishing approaches and (3) feedback approaches. These three categories are widely applied in airline planning for the following five major sub problems:

- Schedule design problem (SDP). Establish flight timetable based on market.
- Fleet assignment problem (FAP). Assign proper fleet type to flights.
- Aircraft routing problem (ARP). Construct a sequence of flights for aircraft.
- Crew scheduling problem (CSP). Assign crew trips to execute flights.
- Tail assignment problem (TAP). Close to day of operation, construct routes for specific aircraft.

A. Background

An overview of different investigations for robust airline planning is illustrated in TABLE I. Domain-specific approaches usually identify a particular feature to approximate robustness in airline planning process. Typical features includes: reasonable allocation of buffer time [3], [4] which is expected to absorb delay propagation; short cycles and less fleet types to provide more opportunities for airline recovery [5], [6]; isolating disruption in a particular airport to protect other flights [7] and evaluation criterion like flight service level [8]. The performance of these approaches mainly depends on their efficacy gap of robustness and can fluctuate among datasets with mark difference.

Propensity-diminishing approaches, on the other hand, combine robustness within model formulations to decrease the probabilty of disruptions and delays occurrance. One representative thought in this aspects is to minimize the expectation of propagated delay by retiming the flight schedule. [9] proposed the delay propagation and rearranged slack time so as to reduce delay along downstream flights. [12] integrated ARP and CSP to capture joint reaction for aircraft and crew in minimizing total delay time. Adopting a new compact formulation, [15] minimized the propagated delay in the weekly line of flight (WLOF) network for ARP and partly extended to TAP. Besides

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TABLE I
SUMMARY OF LITERATURE FOR ROBUST AIRLINE PLANNING

Papers	Approaches			Covering aspects	Robustness feature
	DSA[1]	PDA[2]	FBA[3]		
[5]	✓			ARP	Aircraft swap opportunities
[3]	✓			CSP	Sufficient/tight ground time
[7]	✓			FAP	Hub isolation & short sycles
[6]	✓			FAP	Station purity
[9]		✓		ARP	Retiming to minimize propagated delay
[10]			✓	CSP	Crew switching aircraft delay
[11]		✓		FAP	Retiming & refleeting for demand uncertainty
[8]	✓			SDP	Considering block time uncertainty
[12]		✓		ARP, CSP	Minimizing total propagated aircraft & crew delay
[13]			✓	TAP	Easing recovery process
[14]			✓	TAP	Penalizing maintenance misalignment
[15]		✓		FAP, TAP	WLOF model to minimize propagated delay
[16]		✓		ARP	Extreme-value paradigm
[17]			✓	SDP, FAP	Integrated model under demand uncertainty
[18]	✓			ARP	Retiming with Monte-Carlo-based procedure
[19]		✓		ARP	chance constraint & extreme-value paradigm
[4]	✓			ARP, CSP	Integrated compact model with sufficient buffers

[1] DSA: domain-specific approaches
[2] PDA: Propensity-diminishing approaches
[3] FBA: Feedback approaches

that, [11] solved robust SDP through retiming and refleeting, considering demand uncertainty. [16] presented a extreme-value based framework for robust ARP where flight delays lied in a correlated uncertainty set. [19] set up new models in extreme-value and chance-constrained paradigms to minimize delay for ARP which had the potential for more general resource-allocation problems.

Within the third category, feedback approaches utilize the weight information via recourse (second stage) problem under simulated scenarios. [10] proposed a model for this approach. Specifically, robust CSP was solved by minimizing delay of short connection within a set of scenarios. [13] presented a recoverable TAP model so as to develop a schedule that is recoverable with limited efforts. Similarly, [14] constructed single day routes for aircraft under the recoverable robustness framework. By penalizing the under supply of routes ending at maintenance stations, they got maintenance plan which is recoverable from disruption. In [17], the author took market demand uncertainty into account and proposed a two-stage mixed integer non-linear model for SDP and FAP jointly. A key feature of the feedback approaches lies in its dynamic adjustment of scheduled plan according to the recourse subproblem's feedback on reaction to the plan. Compared with the other two approaches, feedback approaches are able to build robustness in a more direct and realistic way.

B. Contributions

In this paper, our focus is on the tail assignment step of airline planning process. Although robust TAP has been studied previously, we observe that some aspects ought to be further addressed. The first aspects is computational tractability. Because the feedback approaches often refer to many recourse subproblems for different scenarios, the corresponding solution algorithms contain 2 or more stages through benders decomposition. While existing literature deal datasets with comparatively smaller size or shorter timespan, it is desirable to have more efficient solution algorithms to tackle this complex model. The second aspect is realized operational considerations. Among most existing studies in TAP related fields, airport capacity, maintenance misalignment instead of delay are not well studied while they can be quite crucial as maintenance events tend to be vulnerable towards disruptions and provide little recourse within the whole schedule. In addition, the trade-off between maintenance misalignment and its flexibility for possible swap is expected to better handled to save recovery cost. In view of these not fully explored aspects, we summarize our contributions of this work as follows. First, we establish a column-and-row generation heuristic to solve the stochastic tail assignment problem with recovery initiative where improved column generation and benders decomposition algorithms are adopted to render satisfying results in short time. In contrast to traditional approaches, our approach can boost the computation performance by a factor of 7 at least. Second, operational insights are derived for airport capacity and maintenance trade-off. Aircraft routes are generated with lower cost and make allowance to recovery actions (i.e. aircraft swap, flight cancellation and maintenance swap). Our recovery model is an extension of [20] and is able to generate recovery plan efficiently. Because we use a benders decomposition framework, this subproblem model explicitly considers deviation from TAP model's solution in model formulations to establish the benders cuts.

C. Outline

The paper is organized as follow: Section II illustrates the mathematical model in detail with emphasis on explaining how benders decomposition and column generation works on the original deterministic model. Section 3 presents the overall improved solution techniques towards this complex set of models. Section 4 reports on the extensive computational experience and related features obtained. Finally, Section 5 makes conclusions and indicates the next research steps.

II. The Mathematical Model

The stochastic tail assignment under disruption recovery initiative (STAPDRI) can be defined as follows. Given a close to operation schedule(4-7 days), the objective of STAPDRI is to assign routes to individual aircraft that minimizes the overall operational cost and disruption cost subject to 5 operational constraints [?]. (1) Flight coverage: each flight should be covered by one aircraft.(2) Equipment continuity: an aircraft can depart from an airport if it arrives at this airport before. (3)Initial conditions: every aircraft must depart from its initial location. (4) Turn time constraints: a minimum time has to be reserved between 2 consecutive flights. (5) Maintenance check: mandated by FAA, aircraft type A checks should be guaranteed at maintenance station after operating for a fixed time period (flying tions of this work as follows. First, we establish a column-and-row generation heuristic to solve the stochastic tail assignment problem with recovery initiative where improved column generation and benders decomposition algorithms are adopted to render satisfying results in short time. In contrast to traditional approaches, our approach can boost the computation performance by a factor of 7 at least. Second, operational insights are derived for airport capacity and maintenance trade-off. Aircraft routes are generated with lower cost and make allowance to recovery actions (i.e. aircraft swap, flight cancellation and maintenance swap). Our recovery model is an extension of [20] and is able to generate recovery plan efficiently. Because we use a benders decomposition framework, this subproblem model explicitly considers deviation from TAP model's solution in model formulations to establish the benders cuts. time, elapsed time etc). To obtain robust solution, STAPDRI deal with tail assignment problem and aircraft recovery problem(ARP) together using stochastic programming framework through establishing representative scenarios to simulate realized airline operation routine or emergencies.

The classical flight string model introduced by [21] and a flight connection network, where every flight serves as a vertex in the network and is connected with other vertices if the aforementioned operational constraint(2)-(4) are satisfied, are used to develop our STAPDRI model. To facilitate the discussion, we define the following notations in Table II

TABLE II
NOTATION USED FOR EXPRESSING THE MODEL

Parameters	Description
F	the set of flights, $i \in F$
P	the set of aircraft, $j \in P$
D	the set of days of planning $d \in D$
M	the set of maintenance stations, $m \in M$
S	the set of scenarios, $s \in S$
R^p	the set of routes for aircraft p, $r \in R^p$
R^p_s	the set of routes for aircraft p in scenario s, $r \in R^p_s$
C_{md}	the maintenance capacity of airport m on day d
a^1_{ir}	1 if flight i is included in route r
a^2_{mdr}	1 if route r visits maintenance station$m$ at day$D$
w_s	weight coefficient for subproblem in scenario s
d^s_i	the delay cost for flight i in scenario s
g^1	the cost for flight deviation
g^2	the cost for maintenance misalignment
Decision Variables	**Description**
x_{rp}	1 if aircraft p executes route r
x^s_{rp}	1 if aircraft p executes route r in scenario s
y^s_i	1 if flight i is cancelled in scenario s
$\alpha^{s+}_{ip}, \alpha^{s-}_{ip}$	indicate deviation of flight i executed by aircraft p in scenario s
$\beta^{s+}_{mdp}, \beta^{s-}_{mdp}$	indicate deviation of the number of times aircraft p visits station m for day d in scenario s
ϕ_s	cost of subproblem in scenario s

A. Deterministic Model

With these parameters and variables, our STAPDRI model is formulated deterministically as follows:

$$\min \sum_{rp} c_{rp} x_{rp} + \sum_{s \in S} w_s \phi_s \tag{1}$$

$$\phi_s = \min \sum_{r \in R} \sum_{p \in P} c^s_{rp} x^s_{rp} + \sum_{i \in F} d^s_i y^s_i + \sum_{i \in F} \sum_{p \in P} g^1(\alpha^+_{ip} + \alpha^-_{ip}) + \sum_{m \in M} \sum_{d \in D} \sum_{p \in P} g^2 (\beta^+_{mdp} + \beta^-_{mdp}) \tag{2}$$

$$s.t. \sum_{p \in P} \sum_{r \in R^p} a^1_{ir} x_{rp} = 1, \qquad \forall i \in F \tag{3}$$

$$\sum_{r \in R^p} x_{rp} \leq 1, \qquad \forall p \in P \tag{4}$$

$$\sum_{p \in P} \sum_{r \in R^p} a^2_{mdr} x_{rp} \leq C_{md}, \qquad \forall m \in M, d \in D \tag{5}$$

$$\sum_{p \in P} \sum_{r \in R^p_s} a^1_{ir} x^s_{rp} + y^s_i = 1, \qquad \forall i \in F, s \in S \tag{6}$$

$$\sum_{r \in R^p_s} x^s_{rp} \leq 1, \qquad \forall p \in P, s \in S \tag{7}$$

$$\sum_{p \in P} \sum_{r \in R^p_s} a^2_{mdr} x^s_{rp} \leq C_{md}, \qquad \forall m \in M, d \in D, s \in S \tag{8}$$

$$\alpha^{s+}_{ip} - \alpha^{s-}_{ip} + \sum_{r \in R^p_s} a^1_{ir} x^s_{rp} = \sum_{r \in R^p_s} a^1_{ir} x_{rp}, \quad \forall i \in F, p \in P, s \in S \tag{9}$$

$$\beta^{s+}_{mdp} - \beta^{s-}_{mdp} + \sum_{r \in R^p_s} a^2_{mdr} x^s_{rp} = \sum_{r \in R^p_s} a^2_{mdr} x_{rp}, \quad \forall m \in M, d \in D, p \in P, s \in S \tag{10}$$

$$x_{rp} \in \{0,1\}, x^s_{rp} \in \{0,1\}, y^s_i \in \{0,1\}, \phi_s \geq 0 \tag{11}$$

$$\alpha^{s+}_{ip} \geq 0, \alpha^{s-}_{ip} \geq 0, \beta^{s+}_{mdp} \geq 0, \beta^{s+}_{mdp} \geq 0 \tag{12}$$

The objective function of Equation(1) is the sum of flight assignment cost (i.e. fuel cost and maintenance cost) along with a weighted sum of aircraft recovery cost among S scenarios. To be specific, the recovery cost is comprised of flight delay d^s_i, deviation from the original schedule. By using variables

α^{s+}, α^{s-}, differences in one aircraft's assigned flights can be observed and penalized. In the same way, misalignment of aircraft *p*'s maintenance on station *m*, day *d* is expressed with variables β^{s+}, β^{s-}.

Constraints(3) are the flight coverage constraints which means each flight is covered by one aircraft route only. The constraints in Constraints(4) ensure that each aircraft can choose at most one route to fly. Resource constraints(5) place a limitation on the number of maintained aircraft in a maintenance station according to the capacity.

On the side of the aircraft recovery stage, the superscript *s* denotes the recovery scenario *s* that the model belongs to. Constraints (6) indicate that each flight is either canceled or covered by one route in scenario *s*. The restriction on the number of available aircraft and airport capacity is described in constraints(7-8) corresponding to constraints(4-5). Constraints set (9-10) capture the deviation of flight and maintenance from original plan x_{rp}. As we add the deviation decision variables α, β with positive objective value, the optimal solution requires these variables to be strictly constrained by their lower bound. Moreover, the upper bound of deviation variables are restricted to 1 in the optimal solution.

From an intuitive view, the deterministic model of STAPDRI is a large-scale mixed integer program that solves the TAP and multiple ARPs simultaneously, given a bunch of scenarios. Such a model is too complex to be solved directly, to ease the computational burden and tackle some real-life problem, decomposition techniques shall be applied. As the model is amenable to the L-shaped method in stochastic programming [22] and the number of feasible flight routes can increase exponentially with the problem size, so enumeration is out of consideration. In this paper, benders decomposition and column generation are adopted to form our solution algorithm. The following parts deal with the basic formulation of benders decomposition and column generation. Improving skill is introduced in section III.

B. Benders decomposition

Benders decomposition has been successfully applied to combinatorial optimization problems. Its main idea is to decompose a complex model into a master problem (MP) and sub-problem (SP) [23]. For our STAPDRI model, the decomposition problem is clear that constraints related to variables x_{rp} form the master problem while other decision variables comprise subproblems. The mathematical formulations for primal subproblems (PSP) and the master problem(MP) are given as follow

(PSP)

$$\phi_s = \min \sum_{r\in R}\sum_{p\in P} c^s_{rp} x^s_{rp} + \sum_{i\in F} d^s_i y^s_i + \sum_{i\in F}\sum_{p\in P} g^1(\alpha^+_{ip} + \alpha^-_{ip}) + \sum_{m\in M}\sum_{d\in D}\sum_{p\in P} g^2(\beta^+_{mdp} + \beta^-_{mdp}) \quad (13)$$

$$s.t. \sum_{p\in P}\sum_{r\in R^p_s} a^1_{ir} x^s_{rp} + y^s_i = 1, \qquad \forall i \in F \quad (14)$$

$$\sum_{r\in R^p_s} x^s_{rp} \le 1, \qquad \forall p \in P \quad (15)$$

$$\sum_{p\in P}\sum_{r\in R^p_s} a^2_{mdr} x^s_{rp} \le C_{md}, \qquad \forall m \in M, d \in D \quad (16)$$

$$\alpha^{s+}_{ip} - \alpha^{s-}_{ip} + \sum_{r\in R^p_s} a^1_{ir} x^s_{rp} = \sum_{r\in R^p_s} a^1_{ir} x^*_{rp} \qquad \forall i \in F, p \in P \quad (17)$$

$$\beta^{s+}_{mdp} - \beta^{s-}_{mdp} + \sum_{r\in R^p_s} a^2_{mdr} x^s_{rp} = \sum_{r\in R^p_s} a^2_{mdr} x^*_{rp} \qquad \forall m \in M, d \in D, p \in P \quad (18)$$

$$x^s_{rp} \in \{0,1\}, y^s_i \in \{0,1\}, \phi_s \le 0 \quad (19)$$

$$\alpha^{s+}_{ip} \ge 0, \alpha^{s-}_{ip} \ge 0, \beta^{s+}_{mdp} \ge 0, \beta^{s+}_{mdp} \ge 0 \quad (20)$$

The optimal solution of the master problem is given as fixed input x^* to primal subproblems for further optimization. As the subproblem's constraints can be always satisfied by setting $y^s_i = 1, \forall i \in F$, the solution status of PSP is always feasible. So, after finding the optimal solution of PSP, optimal benders cuts can be generated for the master problem. Consider the linear relaxation of PSP. Let $(\theta^s_i, \gamma^s_p, \mu^s_{md}, \delta^{1s}_{ip}, \delta^{2s}_{mdp})$ be the dual variable of constraints(14)-(18) respectively. Denote the set of benders cuts as Ω. Given subproblem *s* and $w \in \Omega$ the derived optimal benders cuts can be expressed as constraints(22). These constraints can be understood as an estimation of PSP's objective function from the view of weak duality theory. Through iterating between master problem and subproblem, an optimal solution can be reached by checking the gap of lower bound and upper bound.

(MP)

$$\min \sum_{rp} c_{rp} x_{rp} + \sum_{s\in S} w_s \phi_s \quad (21)$$

$$s.t. (3) - (5)$$

$$\phi_s \ge \sum_{i\in F} \theta^{sw}_i + \sum_{p\in P} \gamma^{sw}_p + \sum_{m\in M}\sum_{d\in D} C_{md}\mu^{sw}_{md} + \sum_{p\in P}\sum_{r\in R^p}\left(\sum_{i\in F} a^1_{ir}\delta^{1sw}_{ip} x_{rp} + \sum_{m\in M}\sum_{d\in D} a^2_{mdr}\delta^{2sw}_{mdp} x_{rp}\right) \qquad \forall s \in S, w \in \Omega \quad (22)$$

C. Column generation

In this paper, we use column generation to solve our master problem(TAP) as well as subproblems(ARP). This methodology avoids enumerating decision variables and is efficient in solving a large-scale optimizing problem. Within each iteration of column generation, the problem's LP relaxation is first

Algorithm 1 Multi-label shortest path algorithm for TAP

Input Connection Network *G(V,A)*, Dual values (π_i, π_a, π_{md})
Initialize the source node's label pool as $[(-\pi_a, 0, 0)]$
for each node *n* in G's topological sort **do**
  for each label *l* in node *n*'s label pool **do**
    for each node n_2 such that $(n, n_2) \in A$ **do**
      $l_{12} = (c_n - \pi_n, t_n, t_n + t_{n,n_2})$
      if the node is a flight **then**
        Generate new label $l_2 = l + l_{12}$
      end if
      if the node is a maintenance **then**
        Generate new label $l_2 = (l[0], 0, 0) + l_{12}$
      end if
      if l_2 valid, not dominated
        Insert l_2 in n_2's label pool
        Remove labels in n_2 dominated by l_2
      end if
    end for
  end for
end for
Output Sink node's label set

solved to optimal. Then, based on the dual information, a pricing subproblem is invoked to check whether variables with negative reduced cost(if it is a minimizing problem) exist. Then add this particular variable to the problem or terminate the process. The pricing problem in this context is a shortest path problem with resource replenishment to find a most negative flight route and meet up with the 5 operational constraints.

As for the TAP, let π_i be the dual variable of constraint(3), π_a be the dual variable of constraint(4), and π_{md} the dual variable of constraint(5). Then the reduced cost $\overline{c}_{rp}$ of route *r* is

$$\overline{c}_{rp} = c_{rp} - \sum_{i \in F} \pi_i a^1_{ir} - \pi_a - \sum_{m \in M} \sum_{d \in D} \pi_{md} a^2_{mdr} \tag{23}$$

Similarly. the reduced cost $\overline{c}^s_{rp}$ of route *r* in scenario *s* in ARP is expressed as follow:

$$\begin{aligned} \overline{c}^s_{rp} = c^s_{rp} - \sum_{i \in F} \theta^s_i a^1_{ir} - \gamma^s_p - \sum_{m \in M} \sum_{d \in D} \mu^s_{md} a^2_{mdr} \\ - \sum_{p \in P} \sum_{r \in R^p} \left(\sum_{i \in F} \delta^{1s}_{ip} a^1_{ir} + \sum_{m \in M} \sum_{d \in D} \delta^{2s}_{mdp} a^2_{mdr} \right) \end{aligned} \tag{24}$$

As described at the beginning of section II, we model our pricing problem on a directed connection network *G=(V, A)*. In the network, vertices *V* are flights and maintenance events. The arcs *A* exists if the arrival airport of a previous node is identical to the current node's departure airport(maintenance vertices connect from and to the same airport) while minimum turn time(maintenance time) need to be reserved as well. To find such a negative cost path within *G*, a multi-label shortest path algorithm is used and summarized in Algorithm 1.

As the connection network is an acyclic graph, the main loop of this algorithm checks every vertex in topological order. Initializing the source node with a label *l* to track the changes in reduced cost, flying time after last maintenance check and elapsed time (i.e. flying time and ground time) since last maintenance check. A new label is generated and validated by making sure not being dominated by other existing labels or exceeding maintenance rules. Analogically, we apply the multi-label setting algorithm to solve ARP by adding one new label elements as delay time and calculating the reduced cost based on delay time.

III. Accelerating techniques

Although section II has introduced two decomposition skills applied to our STAPDRI model. Their basic implementations still suffer from slow convergence. In this section, accelerating techniques are introduced for both benders decomposition and column generation. Finally, an overall solution framework is presented with the two improving algorithms.

A. Improving bender decomposition

The speed of benders decomposition is closely related to the strength of the benders cuts generated. To cope with the degeneracy of subproblems, where multiple optimal dual solutions exist, [24] selected the most dominated cuts in terms of pareto optimality. For the sake of simplicity, we rewrite the MIP as $\min c'x + d'y : A_1 x + A_2 y \geq b, x \geq 0, y \in \mathbf{Y}$. All vectors and matrices are of suitable size. Then introducing the dual variable α to express the dual problem $\max \alpha'(b - A_2\overline{y}) : A'_1\alpha \leq c, \alpha \geq 0$. So Magnanti-Wong made up an auxiliary problem to get that pareto-optimal cut. The auxiliary problem get the form as: $\max \alpha'(b - A_2\hat{y}) : A'_1\alpha \leq c, \alpha'(b - A_2\overline{y}) = Q(\overline{y})\alpha \geq 0$. where $Q(\overline{y})$ is the optimal objective value of the regular dual subproblem. In order to circumvent the numerical problem of Magnanti-Wong method, [25] provided a more practical definition of Magnanti-Wong point and thus changed the direction of cuts with Pareto-optimal SP: $\max \alpha(b - By^{MW} : A'\alpha \leq c, \alpha \geq 0)$. Here y^{MW} is the Magnanti-Wong point which can be updated during every iteration: $y^{MW} = (1 - \lambda)y^{MW} + \lambda\overline{y}$. If $\overline{y}$ leads to a bounded solution of SP, then 0.5 is the most effective value for λ. Recall that our ARP model will always be feasible for a given TAP model solution, the Magnanti-Wong point in this case can be obtained from a convex combination: $x^{MW}_{rp} = 0.5 * \overline{x}_{rp} + 0.5 * 0$. For more researches on benders decomposition, one may refer to [26] on more comprehensive view.

B. Improving column generation

From a wide range of research, the most significant speed-ups of column generation come from accelerating the pricing subproblem [27]. In our case, the multi-label setting algorithm takes up a great amount of computational effort and is thus the bottleneck for our overall solution process. Because pre-processing is proven to be quite effective in boosting the multi-label setting algorithm [28], we develop our preprocessing

Algorithm 2 Preprocessing algorithm

Input Connection Network *G(V,A)*, Dual values
Initialize $LB = -\infty, UB = 0, incumbentpath = \emptyset$
Run FSPC and BSPC to get $\eta_{si}^{fc}, \eta_{it}^{bc}, path_{si}^{fc}, path_{si}^{bc}$
$LB = \eta_{st}^{fc}$
if $valid_s^{fc}t$ **then**
    $UB = \min(UB, \eta_{st}^{fc})$
    $incumbentpath = path_{st}^{fc}$, go to **Output**
end if
for j in Resource set **do**
    Run FSPRj, BSPRj to get $\eta_{si}^{frj}, \eta_{si}^{brj}, path_{si}^{frj}, path_{si}^{brj}$
    if $valid_{st}^{frj} \& UB > \eta_{st}^{fc}$ **then**
        $UB = \eta_{st}^{fc}, incumbentpath = path_{st}^{frj}$
    end if
end for
for node $n \in V$ **do**
    if $valid_{sn}^{f*} \& valid_{nt}^{b*} \& UB > \eta_{st}^{f*} + \eta_{nt}^{b*}$**then**
        $UB = \eta_{st}^{f*} + \eta_{nt}^{b*}$
        $incumbentpath = path_{sn}^{f*} + path_{nt}^{b*}$
    end if
end for
Output LB, UB, η_{it}^{bc}
* denotes *c, rj*

method to provide relevant information for the multi-label setting solver to accelerate the column generation. Our method is established on the work of [29].

Our method starts from the shortest path algorithms from the source node to all other nodes and then run it again backwardly from sink node to other nodes. We refer to this method as forward shortest path on cost(FSPC) and backward shortest path on cost(BSPC). Meanwhile, accumulated resources(flying time etc.) since last maintenance along with validity check(if the path violates resource limitation) have to be recorded. If the path from the source node to sink node exist and meet up with maintenance rules, we accepted as most negative reduced cost path and no need for multi-label shortest path algorithm. Otherwise, we need to run 4 more shortest path algorithm (correspond to 2 resources) on minimizing flying time or elapsed time both forward and backward (refer to FSPR1, BSPR1, FSPR2, BSPR2 respectively) and update the lower bound(LB) as the reduced cost of FSPC from source node s to sink node t: η_{st}^{fc}. Check these resource shortest paths' reduced cost $\eta_{st}^{fr1}, \eta_{st}^{fr2}$ with feasibility check to update the upper bound(UB). Finally, concatenate these forward paths and backward paths on potential vertex with updated UB after checking feasibility. The algorithm terminates with global LB, UB and recorded shortest reduced cost/resource consumption from every node to the sink node $t(\eta_{it}^{bc}, \eta_{it}^{br1}, \eta_{it}^{br2})$, as described in Algorithm 2. Note that we do not concatenate all forward paths with the backward path but heuristically choose to concatenate paths generated, for instance, in FSPC with those in BSPR1(BSPR2) because they are more likely to become a feasible paths in terms of resource consumption.

Algorithm 3 Overall solution algorithm

Initialize *UB*=$+\infty$,*LB*=0, *GAP*=$+\infty$,y^{MW}.
while *GAP>0* **do**
    LB←**Solve** MP (21)
    for scenario $s \in S$
        $PSP^* \leftarrow$ **Solve** PSP (13)-(20)
        $\lambda^* = 0.5$
        Update M-W point
        Add optimal benders cuts (22) to MP
        Solve Benders Pareto-Optimality PSP
        Add optimal cuts (22) to MP
    end for
    Update *UB*
    GAP=$\frac{(UB-LB)}{LB}$
end while
Reintroduce integrity requirements for MP
Solving MP using dive-and-price

Solution results from Algorithm 2 can help to accelerate multi-label setting algorithm by removing labels whose reduced cost (resource consumption) has or will be larger than current UB (resource limitations) while not removing any label that affects the proven optimality.

C. The overall algorithm

In order to solve STAPDRI with the decomposition techniques discussed above, we developed an overall solution framework for our model. As flight string model for TAP usually generates a tight linear relaxation. The overall algorithm will solve the linear relaxation of STAPDRI and introduce integrality constraint to the TAP model afterwards. Instead of solving the integral TAP model with precise but time-consuming branch and price method(with follow on branching). We adopt a diving heuristic without backtracking to get the final result. This diving heuristic is also utilized in [28]. The whole algorithm is illustrated in Algorithm 3.

The algorithm starts with a linear relaxation of the master problem(MP), and reintroduce the integrity requirements after the relative gap between upper bound(UB) and lower bound(LB) meet a given value. The MP and primal subproblem(PSP) are all solved with Algorithm 1 and Algorithm 2. The multi-label setting algorithm is only called when the preprocessing algorithm can't find negative variables or optimality need to be proved. After that, the MIP MP is solved with dive-and-price algorithm using a branching scheme called follow-on branching [30]. This strategy differs from traditional variable fixing and is widely used in scheduling problem which avoid unbalanced search tree.

IV. Computational Experiments

A. Data description

To validate the performance of the proposed column and row generation solution framework, computational experiments are carried out and the results are illustrated in this section. The solution framework is programmed in Python and run on a laptop with 2.5GHz Intel i7-6500U CPU and Fedora 27 system. SCIP is called to get the integer solution with a CPLEX 12.6.3 as linear programming solver.

All the tests are carried out in a single thread for ease of comparison. The data set is derived from daily operation data of a China domestic low cost air carrier. Based on this, 4 test instances are extracted and prepared. The detailed information of these instances are shown in TABLE III. Information on flights and fleet are derived from published schedule while the maintenance stations are derived using an analytical hierarchy process(AHP).

TABLE III
CHARACTERISTICS OF TEST CASES

Test cases	Flights	Fleet size	Connections	Airports	Maint stations
1	53	5	428	21	4
2	77	7	517	23	5
3	104	7	853	26	6

To better simulate the circumstance of daily operation, we generate 56 scenarios. The specifics of these scenarios are presented in TABLE IV. These scenarios cover common operational situations include flight delay, airport closure, reduction in maintenance capacity and Aircraft-on-Ground(AOG). Specifically, delay is classified as slight, moderate and severe according to three time minutes range $[15, 30), [30, 60), [60, 180)$ respectively. To generate these delay scenarios, we select delay values randomly from historical data. Because the data is extracted from a summer schedule. So, airports are exposed to the risk of afternoon closure due to thermal thunderstorm. While fog is a comparatively less contributor to the morning closure. We assume the airport closure scenarios happen in the early afternoon. Also, to reveal the impact of maintenance capacity reduction, we apply a sharp drop on maintenance capacity (by half) in one operation day. Lastly, it is common in operation that an aircraft experiences an unexpected mechanical failure which requires the aircraft to receive mending at maintenance stations. This is referred to Aircraft-on-Ground (AOG) in the airline industry. In our experiments, AOGs take place for 2 hours and during which the aircraft can not serve any flight.

B. Numerical results

In this part, we report the computation results of STAPDRI for all eight instances. Because the mathematical model is of large-scale and complexity, we proposed our solution algorithm and corresponding enhancement in the last two sections. Two improvements (the pareto optimal cuts and preprocessing algorithm) are implemented and compared with implementations where only one or no enhancement happens. In TABLE V, the different running time results are defined as (1) Basic: using the standard benders decomposition and column generation. (2) All enhancements: including pareto optimal cuts and preprocessing algorithm.

The objective values of both linear relaxation solutions and integer solutions are also reported with the corresponding LP relaxation degree. To better represents realized airline operational situations. We utilize Base of aircraft data family (BADA2) to calculate the fuel consumption cost for specific aircraft type on different cruising flight levels (i.e. simulating short/long range). The average maintenance cost is roughly estimated using statistics from IATA. As delay cost parameters in ARP vary from airlines to airlines, we select two penalty cost values for delay to analyze its impact on the independent delay and propagated delay.

From the computation result, we observed a very long run time for instances without enhancements. In contrast, pareto optimal cuts along with preprocessing boost the performance very much, reducing the running time by a factor of 7 at least. As the computation time depends heavily on the pricing subproblem and the benders decomposition's iterations, our improved solution usually takes much fewer iterations to converge. Also, the preprocessing algorithm for ARP is observed to frequently find the most negative path for pricing. Thus, the improved algorithm tends to take advantages over average solution methodology. Moreover, as the last column in TABLE V indicates, the linear relaxation of our STAPDRI model is quite tight, with a gap of nearly 0.1% between LP solutions and IP solutions. Which motivates us to use a quick diving heuristic for final integer solution.

C. Effects of stochastic solutions

Comparing the result of STAPDRI with that of deterministic TAP alone, we find much improvement margin in the individual ARP objective value across all the scenarios. Take the test case 3 for instance (delay cost is 20/min), we make a comparison between the final objective values of ARP models for all 56 scenarios and those from a deterministic TAP solution. which is shown in Fig 1. As explained in the figure, STAPDRI has the advantage to produce robust solution's whose weighted ARP objective values will be no worser than that of the solution to the model used in deterministic TAP. Recall we category the 56 scenarios into 6 classes. The first three classes(30 cases shown in subfigure a) relating to departure delay constitute a larger cost component because there is much more independent delay at each flight leg. In contrast, airport closure, maint capacity reduction, and AOGs are more prone to cause maintenance misalignment and deviations to planning schedule. When solving ARP, total delay including independent delay and propagated delay are aimed to be minimized. We adopt the formulations in [9] to calculate propagated delay:

$$PD_{ij} = \max(TAD_i - Slack_{ij}, 0)$$

Thus, the propagated delay *PD* from flight *i* to *j* is decided by both total arrival delay *TAD* and slack time *Slack*. In our experiment robust solutions, aircraft have been assigned to flight strings with less probability of propagated delay. This is achieved by leaving more slack time for flights with a long delay time or under the impact of disrupted airports.

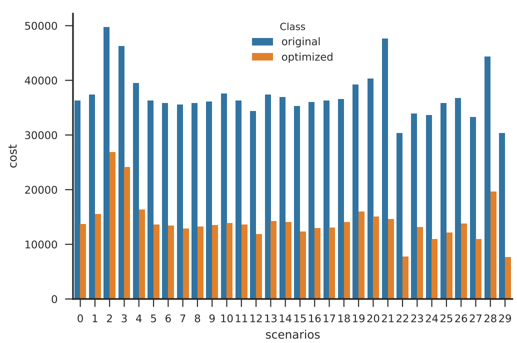
Another important part of our analysis is determining the trade-off between explicit aircraft assignment cost and implicit predicted delay costs. To have a basic understanding of how

TABLE IV
OPERATION SCENARIOS USED

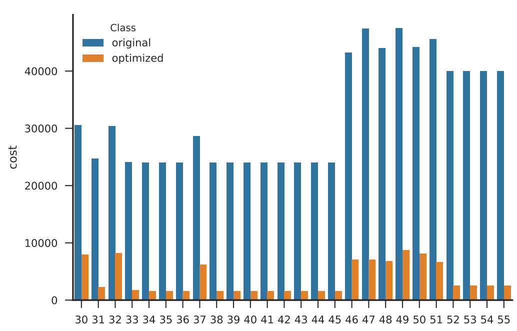
Type	**Affected**	**Scenarios**
Slight flight delay	One scenario for 30% flight legs	0-9
Moderate flight delay	One scenario for 15% flight legs	10-19
Severe flight delay	One scenario for 5% flight legs	20-29
Airport closure	One scenario for each major airport	30-39
Capacity reduction	One scenario for each maintenance station	40-45
Aircraft-on-Ground	One scenario for each aircraft	46-55

TABLE V
COMPUTATION RESULTS OF STAPDRI WITH ENHANCEMENTS

Test case	Delay cost	Running time		LP obj	IP obj	Relaxation
		Basic	All enhancements			
1	20	1052.75	66.65	365282.06	365562.06	0.076
	80	1242.50	83.47	376475.02	376752.40	0.073
2	20	1951.39	143.48	490714.95	490731.21	0.004
	80	1686.80	133.80	504852.02	505432.40	0.115
3	20	3387.51	342.66	688098.03	688444.14	0.050
	80	2318.18	297.67	729977.51	729977.51	0.0



(a)scenarios 0-29



(b)scenarios 30-55

Fig. 1. Comparison in ARP objective values

our delay penalty cost parameter affect this trade-off, we analyze the stochastic solution using different delay cost parameter. TABLE VI demonstrates the sum of propagated delay and total assignment cost. For all our experiments, the delay cost parameter has no effects on the final optimal propagated delay and assignment cost. This indicates our model is quite robust towards different penalty coefficient which eases the difficulty in quantifying the delay cost.

TABLE VI
DELAY COST EFFECTS ON SOLUTIONS

Test case	Delay cost	Propagated delay(s)	Assignment cost
1	20	Is this	361600.00
	80	table necessary?	361600.00
2	20	since the	485498.23
	80	results are the same	485498.23
3	20	5082	678767.68
	80	5082	678767.68

D. *Comparing airline's original plan*

From the comparison above, STAPDRI takes obvious advantageous over deterministic TAP model. In this section, we would like to figure out the strategy used in airline to arrange their schedules. Because degeneracy is common in airline optimization problem that many feasible solutions attain the optimal solution at the same time. Directly comparing the optimal solution with airline's original plan is unreasonable. Instead, the weighted costs of ARP subproblems and other statistics are used to indicate the operation pattern in airline and potential benefits of our model.

Specifically, airline's original timetable is given as input x_{rp} to solve every subproblems. The weight sum of these recovery cost are thus calculated and shown in TABLE VII. The coefficient of delay cost is set to 20, both numeric values and increasing rates are included.

As is clearly illustrated in the table, the airline's original plan has already contains some robustness in constrast to the solution from deterministic TAP model. For test case 3, the

TABLE VII
COMPUTATION STATISTICS WITH WEIGHTED SUM OF ARP

Test case	Original plan	TAP solution	STAPDRI solution
1	4699.29	6163.56 (+31.16%)	3873.50 (-17.57%)
2	9099.29	7211.67 (-20.74%)	4987.91 (-45.18%)
3	7657.14	47790.0 (+524.12%)	7642.63 (-0.19%)

original plan is close to the optimal solution found by our STAPDRI model with a gap smaller than 0.2%. But in test case 2, even TAP solution has a 20 percentage superiority over the airline's solution. Currently, the airline's flight schedule is mainly determined by flight dispatcher from Airline Operation Center (AOC) and staffs from Maintenance Department. While historical operation statistics are taken into consideration, they decide the final aircraft paths and relating buffer times empirically. Flights departing from or arriving at congested airports will be given more buffer times than average airports. On the other hand, short flight rotations (an aircraft starts and ends at the same airport) are usually adopted to ensure flow balance at airports but reduce flexibility during disruption or delay circumstances. Our STAPDRI models, on the contrary, is capable to devise good enough robust solution if scenarios are devised with representativeness. In addition, [31] indicated that some airlines are willing to experience delay in trade of a shorter schedule block time. The STAPDRI can trade-off the overall cost and recovery expense to some extend from the computational experiences.

V. CONCLUSION

In this paper, we propose a stochastic model for tail assignment problem considering recovery reactions. The proposed model hybrid 2 deterministic models(i.e. TAP and ARP) through a stochastic programming framework. To solve the model with efficiency, we propose an improved solution algorithm that combines benders decomposition with column generation. Our experiments show that the model generates tight LP relaxation and can be solved quickly using the presented algorithm. The value of considering robust TAP is also demonstrated from the gap between the deterministic solution and stochastic solution.

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REFERENCES

[1] K. Gopalakrishnan and H. Balakrishnan, "A comparative analysis of models for predicting delays in air traffic networks." ATM Seminar, 2017.

[2] B. of Transportation Statistics, "Year-to-date airline on-time performance for each month."

[3] M. Ehrgott and D. M. Ryan, "Constructing robust crew schedules with bicriteria optimization," *Journal of Multi-Criteria Decision Analysis*, vol. 11, no. 3, pp. 139–150, 2002.

[4] M. B. Ahmed, F. Z. Mansour, and M. Haouari, "Robust integrated maintenance aircraft routing and crew pairing," *Journal of Air Transport Management*, vol. 73, pp. 15–31, 2018.

[5] Y. Ageeva, "Approaches to incorporating robustness into airline scheduling," Ph.D. dissertation, Massachusetts Institute of Technology, 2000.

[6] B. C. Smith and E. L. Johnson, "Robust airline fleet assignment: Imposing station purity using station decomposition," *Transportation Science*, vol. 40, no. 4, pp. 497–516, 2006.

[7] J. M. Rosenberger, E. L. Johnson, and G. L. Nemhauser, "A robust fleet-assignment model with hub isolation and short cycles," *Transportation science*, vol. 38, no. 3, pp. 357–368, 2004.

[8] M. Sohoni, Y.-C. Lee, and D. Klabjan, "Robust airline scheduling under block-time uncertainty," *Transportation Science*, vol. 45, no. 4, pp. 451–464, 2011.

[9] S. Lan, J.-P. Clarke, and C. Barnhart, "Planning for robust airline operations: Optimizing aircraft routings and flight departure times to minimize passenger disruptions," *Transportation science*, vol. 40, no. 1, pp. 15–28, 2006.

[10] J. W. Yen and J. R. Birge, "A stochastic programming approach to the airline crew scheduling problem," *Transportation Science*, vol. 40, no. 1, pp. 3–14, 2006.

[11] H. Jiang and C. Barnhart, "Dynamic airline scheduling," *Transportation Science*, vol. 43, no. 3, pp. 336–354, 2009.

[12] M. Dunbar, G. Froyland, and C.-L. Wu, "Robust airline schedule planning: Minimizing propagated delay in an integrated routing and crewing framework," *Transportation Science*, vol. 46, no. 2, pp. 204–216, 2012.

[13] G. Froyland, S. J. Maher, and C.-L. Wu, "The recoverable robust tail assignment problem," *Transportation Science*, vol. 48, no. 3, pp. 351–372, 2013.

[14] S. J. Maher, G. Desaulniers, and F. Soumis, "Recoverable robust single day aircraft maintenance routing problem," *Computers & Operations Research*, vol. 51, pp. 130–145, 2014.

[15] Z. Liang, Y. Feng, X. Zhang, T. Wu, and W. A. Chaovalitwongse, "Robust weekly aircraft maintenance routing problem and the extension to the tail assignment problem," *Transportation Research Part B: Methodological*, vol. 78, pp. 238–259, 2015.

[16] C. Yan and J. Kung, "Robust aircraft routing," *Transportation Science*, vol. 52, no. 1, pp. 118–133, 2016.

[17] L. Cadarso and R. de Celis, "Integrated airline planning: Robust update of scheduling and fleet balancing under demand uncertainty," *Transportation Research Part C: Emerging Technologies*, vol. 81, pp. 227–245, 2017.

[18] M. B. Ahmed, W. Ghroubi, M. Haouari, and H. D. Sherali, "A hybrid optimization-simulation approach for robust weekly aircraft routing and retiming," *Transportation Research Part C: Emerging Technologies*, vol. 84, pp. 1–20, 2017.

[19] L. Marla, V. Vaze, and C. Barnhart, "Robust optimization: Lessons learned from aircraft routing," *Computers & Operations Research*, vol. 98, pp. 165–184, 2018.

[20] Z. Liang, F. Xiao, X. Qian, L. Zhou, X. Jin, X. Lu, and S. Karichery, "A column generation-based heuristic for aircraft recovery problem with airport capacity constraints and maintenance flexibility," *Transportation Research Part B: Methodological*, vol. 113, pp. 70–90, 2018.

[21] C. Barnhart, N. L. Boland, L. W. Clarke, E. L. Johnson, G. L. Nemhauser, and R. G. Shenoi, "Flight String Models for Aircraft Fleeting and Routing," *Transportation Science*, vol. 32, no. 3, pp. 208–220, 1998.

[22] G. Laporte and F. V. Louveaux, "The integer l-shaped method for stochastic integer programs with complete recourse," *Operations Research Letters*, vol. 13, no. 3, pp. 133–142, 1991.

[23] Y. Xu, W. Dai, X. Sun, and S. Wandelt, "Improved benders decomposition for capacitated hub location problem with incomplete hub networks," in *2017 IEEE Symposium Series on Computational Intelligence (SSCI)*, Nov 2017, pp. 1–8.

[24] T. L. Magnanti and R. T. Wong, "Accelerating benders decomposition: Algorithmic enhancement and model selection criteria," *Operations Research*, vol. 29, no. 3, pp. 464–484, 1981.

[25] N. Papadakos, "Practical enhancements to the magnanti-wong method," *Operations Research Letters*, vol. 36, no. 4, pp. 444–449, 2008.

[26] R. Rahmaniani, T. G. Crainic, M. Gendreau, and W. Rei, "The benders decomposition algorithm: A literature review," *European Journal of Operational Research*, vol. 259, no. 3, pp. 801–817, 2016.

[27] J. Desrosiers and M. E. Lbbecke, *A Primer in Column Generation*, 2005.

[28] S. Ruther, N. Boland, F. G. Engineer, and I. Evans, “Integrated Aircraft Routing , Crew Pairing , and Tail Assignment : Branch-and-Price with Many Pricing Problems,” *Transportation Science*, vol. 51, no. April 2018, 2017.
[29] O. J. Smith, N. Boland, and H. Waterer, “Solving shortest path problems with a weight constraint and replenishment arcs,” *Computers & Operations Research*, vol. 39, no. 5, pp. 964–984, 2012.
[30] D. M. Ryan and B. A. Foster, “An integer programming approach to scheduling,” pp. 269–280, 1981.
[31] L. Hao and M. Hansen, “How airlines set scheduled block times,” in *10th USA/Europe Air Traffic Management Research and Development Seminar, Chicago IL*, 2013.